Fireflies as Role Models for Synchronization in Ad Hoc Networks

Alexander Tyrrell Gunther Auer
DoCoMo Euro-Labs
80687 Munich, Germany
Email: lastname@docomolab-euro.com

Christian Bettstetter
University of Klagenfurt, Mobile Systems Group
9020 Klagenfurt, Austria
Email: Firstname.Lastname@uni-klu.ac.at

***Abstract*— Fireflies exhibit a fascinating phenomenon of spontaneous synchronization that occurs in nature: at dawn, they gather on trees and synchronize progressively without relying on a central entity. The present article[1] reviews this process by looking at experiments that were made on fireflies and the mathematical model of Mirollo and Strogatz [1], which provides key rules to obtaining a synchronized network in a decentralized manner. This model is then applied to wireless ad hoc networks. To properly apply this model with an accuracy limited only to the propagation delay, a novel synchronization scheme, which is derived from the original firefly synchronization principle, is presented, and simulation results are given.**

I. Introduction

In certain parts of South-East Asia alongside riverbanks, male fireflies gather on trees at dawn, and start emitting flashes regularly. Over time synchronization emerges from a random situation, which makes it seem as though the whole tree is flashing in perfect synchrony. This phenomenon [2] forms an amazing spectacle, and has intrigued scientists for several hundred years. Over the years, two fundamental questions have been studied: Why do fireflies synchronize? And how do they synchronize?

The first question led to many discussions among biologists. In all species of fireflies, emissions of light serves as a means of communication that helps female fireflies distinguish males of its own species: the response of male fireflies to emissions from females is different in each species. However it is not clear why in certain species of fireflies, males synchronize. Several hypothesis exist: Either it could accentuate the males rhythm or serve as a noise-reduction mechanism that helps them identify females [3]. This phenomenon could also enable small groups of males to attract more females, and act as a cooperative scheme [3].

Although the reason behind synchronization is not fully understood, fireflies are not the only biological system displaying a synchronized behavior. This emergent pattern is present in heart cells [4], where it provides robustness against the death of one or more cells, and in neurons, where it enables rapid computation [5]. Among humans, synchronization also occurs. For example, women living together tend to synchronize their menstrual periods [6], and people walking next to each other on the street tend to walk in synchrony.

In Section II, we look at experiments that have been made in order to comprehend the synchronization mechanism. In particular, we focus on experiments made on fireflies and how this affected their flashing instants, providing interesting insights to compare nature with mathematical models.

In biological systems distributed synchronization is commonly modeled using the theory of coupled oscillators [7]. For fireflies, an oscillator represents the internal clock dictating when to flash, and upon reception of a pulse from other oscillators, this clock is adjusted. Over time, synchronization emerges, i.e. pulses of different oscillators are transmitted simultaneously. Synchronization in populations of coupled oscillators lies within the field of discrete nonlinear dynamics. A theoretical framework for the convergence to synchrony in fully-meshed networks was published in [1]. This model will be presented in Section III, and will serve as a basis for deriving a suitable synchronization algorithm for wireless systems.

The Mirollo and Strogatz model of [1] has already been applied to wireless networks. One of the first papers to apply the firefly synchronization model to wireless networks was [8]. It utilized the characteristic pulse of Ultra-Wide-Band (UWB) to emulate the synchronization process of pulse-coupled oscillators, and included more realistic effects such as channel attenuation and noise.

To lift the restriction of using UWB pulses and apply the model of [1] to wireless systems, delays need to be taken into account. Both models of [1] and [8] assume that fireflies form a fully-meshed network and communicate through pulses. However pulses are hardly considered for communications in a wireless environment, because they are difficult to detect.

To reflect more realistic effects such as message delay and loss, [9] proposed to synchronize using a low-level timestamp on the MAC layer. The principle is similar to the original firefly synchronization scheme, in the way that each node adjusts its clock when receiving such a timestamp. Because timestamps need to be exchanged, the approach of [9] tries to avoid the ideal case of the Mirollo and Strogatz model where all nodes transmit simultaneously. This case creates too many collisions, which prevails nodes from synchronizing. However, from a physical layer perspective, all nodes transmitting

[1]A preliminary version of this paper was presented at 3rd workshop of the ESF-funded scientific program MiNEMA, Leuven, Belgium, February 2006. The authors would like to acknowledge MiNEMA for their support.

synchronously a common word can help a faraway receiver synchronize and communicate with the rest of the network, because it receives the sum of all transmitted powers (known as the reachback problem). Hence, unlike data transmission, a synchronization process where all nodes transmit the same word is not affected by collisions in a similar way to flooding. Taking advantage of a common synchronization word also helps the network synchronizing quicker.

Therefore our approach, which also integrates realistic effects such as transmission delays, bases the synchronization algorithm entirely on the physical layer. This has several advantages over [9] such as the fact that collisions are in fact a benefit to the scheme and the time to reach a synchronized state is shorter because there is no random backoff. This synchronization strategy, which was first introduced in [10] and is presented in Section IV, combats transmission and processing delays by modifying the intrinsic behavior of a node. Differently from [10], the present paper studies similarities between experiments made on fireflies and mathematical interpretations that have been made, and deepens the analysis.

II. Experiments on Fireflies

Early hypotheses had difficulties explaining the firefly synchronization phenomenon. For example, Laurent in 1917 dismissed what he saw and attributed the phenomenon to the blinking of his eyelids [11]. Others argued that synchrony was provoked by a single stimulus received by all fireflies on the tree [12]. However the presence of a leading firefly or a single external factor is easily dismissed by the fact that not all fireflies can see each other and fireflies gather on trees and progressively synchronize. The lack of a proper explanation until the 1960s is mostly due to a lack of experimental data.

Among early hypotheses, Richmond [13] stated in 1930 what came very close to the actual process: "Suppose that in each insect there is an equipment that functions thus: when the normal time to flash is nearly attained, incident light on the insect hastens the occurrence of the event. In other words, if one of the insects is almost ready to flash and sees other insects flash, then it flashes sooner than otherwise. On the foregoing hypothesis, it follows that there may be a tendency for the insects to fall in step and flash synchronously."

This statement identifies that synchronization among fireflies is a self-organized process, and fireflies influence each other: they emit flashes periodically, and in return are receptive to the flashes of other males.

To understand this process, a set of experiments was conducted by Buck *et al.* [14]. These experiments concentrated on the reaction of a firefly to an external signal depending on when this light is received. Naturally a firefly emits light periodically every 965 ± 90 ms [14], and the external signal changes this natural period.

For the experiments, the firefly was put in a dark room and was restrained from seeing its own flashes. Stimuli were made by guiding 40 ms signals of light from a glow modulator lamp into the firefly's eye via a fiber optics. Responses were recorded and are shown on Fig. 1.

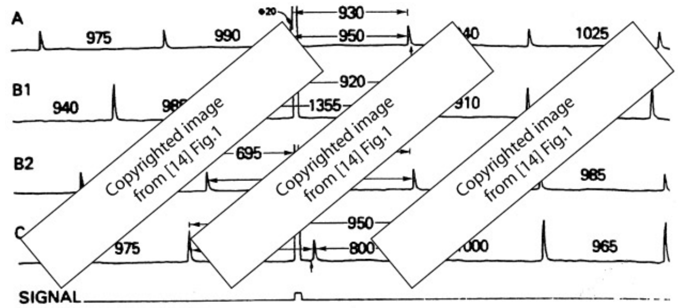

Fig. 1. Experiments by Buck *et al.* (from [14] with kind permission of ©Springer Science and Business Media). Delays are expressed in ms.

From Fig. 1, when an external signal is emitted, three different responses are identified:

- In response A, the artificial signal occurs only 20 ms after the firefly's spontaneous flash. As the following response from the firefly occurs at a normal time of 950 ms, the signal does not seem to have modified the natural response. This corresponds to a refractory period: during this time, the potential of the flash regains the "resting" position and no modification of the internal clock is possible.
- In responses B1 and B2, the signal inhibits the response of the firefly: instead of emitting light after about 960 ms, it delays its response until 920 ms and 940 ms after receiving the signal. Thus successive flashes occur 1355 ms and 1635 ms, which is far more than the natural period.
- In response C, the artificial signal occurs 150 ms before the natural flashing, and does not have any incidence on this flash. This is due to a processing delay in the central nervous system of a firefly, which is equal to about 800 ms [14]. Therefore the external signal influences the following flash, which is advanced by 150 ms. Thus the external signal has an excitatory effect on the response and brings the firefly to flash earlier.

The modified behavior of the firefly depended only on the instant of arrival of the external signal. The responses display both inhibitory (responses B1 and B2) and excitatory (response C) couplings depending on the instant the external flash is perceived. Furthermore a refractory period placed after emission is also present (response A).

In all cases, the external flash only altered the emission of one following flash, and in the following period, nodes regained their natural period of about one second. Variating the amplitude of the input signal also yielded similar results.

For more insights into the phenomenon or firefly synchronization, Chapter 10 in [12] provides a history of studies on fireflies, including early interpretations, and analyzes different experiments including the one presented in this section.

These experiments have helped mathematicians to model fireflies. However proving that synchrony occurs when both inhibitory and excitatory coupling are present has, to our knowledge, not been done yet. Therefore we will concentrate on the existing model of [1], which considers exhitatory coupling and no delays, in order to derive a synchronization algorithm suited for wireless systems.

III. Firefly Synchronization

The internal clock of a firefly, which dictates when a flash is emitted, is modeled as an oscillator, and the phase of this oscillator is modified upon reception of an external flash. In general this type of oscillator is termed *relaxation oscillators*, which are not represented by a typical sinusoidal form but rather by a series of pulses. There is no general model describing this class of oscillators, and some examples include Van der Pol oscillators and integrate-and-fire oscillators [1]. A review of this class of oscillators and their implications can be found in Chapter 1 in [15].

In the remainder of this paper, we will focus on integrate-and-fire oscillators, which are also termed "pulse-coupled oscillators". Pulse-coupled oscillators interact through discrete events each time they complete an oscillation. The interaction takes the form of a pulse that is perceived by neighboring oscillators. This model is used to study biological systems such as heart cells, neurons and earthquakes [5]. This section describes how time synchronization is achieved in a decentralized fashion in a system of N oscillators.

A. Mathematical Model

Each oscillator i, $1 \leq i \leq N$, is described by a state variable x_i, similar to a voltage-like variable in a RC-circuit, and its evolution and interactions are described by a set of differential equations [16]:

$$\frac{dx_i(t)}{dt} = -x_i + I_0 + \sum_{\substack{j=1 \\ j \neq i}}^{N} J_{i,j} \cdot P_j(t) \tag{1}$$

where I_0 controls the period of an uncoupled oscillator and $J_{i,j}$ determines the coupling strength between oscillators. When $x_i = x_{\text{th}}$, where x_{th} is the state variable threshold, an oscillator is said to 'fire': at this instant, its state is reset to zero and it emits a pulse, which modifies the state of other coupled oscillators. The coupling function P_j is defined as a train of emitted pulses:

$$P_j(t) = \sum_m \delta\left(t - \tau_j^{[m]}\right) \tag{2}$$

where $\tau_j^{[m]}$ represents the m^{th} firing time of oscillator j and $\delta(t)$ is the Dirac delta function.

As (1) is not solvable in closed-form for arbitrary N, [1] relies on a discrete approach. To demonstrate that synchrony is always achieved independently of initial conditions, each oscillator is described by a phase function ϕ_i which linearly increments from 0 to a phase threshold ϕ_{th} and periodically fires every T seconds:

$$\frac{d\phi_i(t)}{dt} = \frac{\phi_{\text{th}}}{T} \tag{3}$$

When $\phi_i(t) = \phi_{\text{th}}$, a node resets its phase to 0. If not coupled to any other oscillator, it naturally oscillates and fires with a period equal to T. Fig. 2(a) plots the evolution of the phase function during one period when the oscillator is isolated.

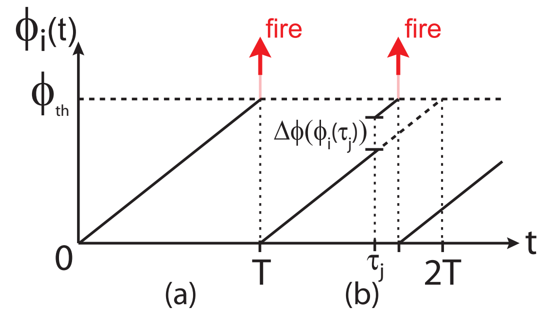

Fig. 2. Time evolution of the phase function

The phase function encodes the remaining time until the next firing, which corresponds to an emission of light for a firefly. The goal of the synchronization algorithm is to align all internal counters, so that all nodes agree on a common firing instant. To do so, the phase function needs to be adjusted.

The phase function dictates when a pulse or a flash is emitted. For a firefly, the natural period T is equal to about one second. In order to simplify the analysis, the Mirollo and Strogatz model does not encompass the clock jitter of $\pm 90\,\text{ms}$ that is experimentally observed (Fig. 1). Therefore in the following, we consider that all nodes have the same dynamics, i.e. clock jitter is considered negligible.

For considerations about clock jitter and frequency adjustment, a different oscillator model was proposed by Ermentrout in [17], where oscillators have different frequencies.

B. Synchronization of Pulse-Coupled Oscillators

Mirollo and Strogatz analyzed spontaneous synchronization phenomena and also derived a theoretical framework based on pulse-coupled oscillators for the convergence of synchrony [1].

When coupled to others, an oscillator is receptive to the pulses of its neighbors. Coupling between nodes is considered instantaneous, and when a node j ($1 \leq j \leq N$) fires at $t = \tau_j$, i.e. $\phi_j(\tau_j) = \phi_{\text{th}}$, all nodes adjust their phase as follows:

$$\begin{cases} \phi_j(\tau_j) = 0 \\ \phi_i(\tau_j) = \phi_i(\tau_j) + \Delta\phi(\phi_i(\tau_j)) \text{ for } i \neq j \end{cases} \tag{4}$$

Fig. 2(b) plots the time evolution of the phase when receiving a pulse. To simplify notations, the parameter m in Eq. (2) is dropped, which is coherent with Fig. 1 where the external signal affected only one following flashing.

By appropriate selection of $\Delta\phi$, a system of N identical oscillators forming a fully-meshed network is able to synchronize their firing instants within a few periods [1]. The phase increment $\Delta\phi$ is determined by the Phase Response Curve (PRC). For their mathematical demonstration, Mirollo and Strogatz derive that synchronization is obtained whenever the firing map $x_i(t) = f(\phi_i(t))$ is concave up and the return map $\phi_i(t) + \Delta\phi(\phi_i(t)) = g(x_i(t) + \epsilon)$, where ϵ is the amplitude increment, is its inverse [1]. The resulting operation

$\phi_i(t) + \Delta\phi(\phi_i(t)) = g(f(\phi_i(t)))$ yields the PRC, and is a piecewise linear function:

$$\phi_i(\tau_j) + \Delta\phi(\phi_i(\tau_j)) = \min\left(\alpha \cdot \phi_i(\tau_j) + \beta, 1\right)$$
$$\text{with} \quad \begin{cases} \alpha = \exp(b \cdot \epsilon) \\ \beta = \frac{\exp(b \cdot \epsilon) - 1}{\exp(b) - 1} \end{cases} \tag{5}$$

where b is the dissipation factor. Both factors α and β determine the coupling between oscillators, and are identical for all. The threshold ϕ_{th} is normalized to 1.

It was shown in [1] that if the network is fully-meshed as well as $\alpha > 1$ and $\beta > 0$ ($b > 0$, $\epsilon > 0$), the system always converges, i.e. all oscillators will fire as one, independently of initial conditions. The time to synchrony is inversely proportional to α.

As it can be observed on Fig. 2(b), detection of a pulse shortens the current period and causes an oscillator to fire early, because $\Delta\phi(\phi_i(t)) > 0, \forall \phi_i(t)$. Compared to the experimental data of Fig. 1, the Mirollo and Strogatz model exhibits only excitatory coupling, and no refractory period is present.

However the main features of the experiments are present: nodes do not need to distinguish the source of the synchronization pulse, and adjust their current phase upon reception of a pulse. The synchronization scheme relies on the instant of arrival of a pulse and receivers adjusting their phases when detecting this pulse.

Applied to wireless systems, this has the advantage that interference and collision is not observed, because a receiver does not need to identify the source of emission. Furthermore two pulses emitted simultaneously can superimpose constructively, which helps a faraway receiver synchronize. This type of spatial averaging has been shown to beneficially bound the synchronization accuracy to a constant, making the algorithm scalable with respect to the number of nodes [18].

IV. Application to Wireless Systems

In wireless systems, different delays need to be taken into account. The algorithm needs to be modified to account for propagation delays, so that the system remains stable. Moreover *long synchronization words* need to be considered for the synchronization scheme, and a receiver requires some decoding delay to properly identify that a synchronization message was transmitted. As these delays affect the achievable accuracy, we will modify the intrinsic behavior of a node, so that high accuracy is regained, and evaluate the novel scheme through simulations.

A. Synchronization through Pulses

Within the field of nonlinear oscillators, it is known that when a delay occurs, even if it is constant between all nodes, then a system of pulse-coupled oscillators becomes unstable, and is never able to synchronize [19]. In wireless systems, even when considering communication through pulses, a propagation delay dependent on the distance between nodes occurs.

If considering a propagation delay between two nodes i and j, denoted by $T_0^{(i,j)}$, then the pulse of i influences j not instantly as before, but after $T_0^{(i,j)}$. If this causes j to reach the threshold and transmit a pulse, then i also increments its phase after $T_0^{(i,j)}$, which can cause it to fire, and so on. If more than two nodes are present in the system, nodes continuously fire.

To avoid this unstable behavior, a refractory period of duration T_{refr} is added after firing: after transmitting its pulse, a node i stays in a refractory state, where $\phi_i(t) = 0$ and no phase increment is possible, and then goes back into the listening state where its phase follows (3). This period is also observed in case A in the experimental data of Fig. 1.

Stability is maintained if echoes are not acknowledged, which translates to a condition on T_{refr}:

$$T_{\text{refr}} > 2 \cdot T_0^{[\text{max}]} \tag{6}$$

where $T_0^{[\text{max}]}$ is the maximum propagation delay between two nodes in the network. With the introduction of the refractory state, the accuracy of the synchronization scheme is equal or smaller to the maximum propagation delay.

B. Transmission Delays in Wireless Systems

The previous scheme implies that nodes communicate through pulses and that a receiver is able to immediately detect a single pulse of infinitely small width, and no decoding is done by the receiver. In a wireless environment solitary pulses are hardly used alone as they are virtually impossible to detect. More realistically a synchronization word is used. In the original firefly synchronization scheme, nodes do not need to distinguish between emitters. Therefore a common synchronization word is broadcasted by all nodes when firing.

The synchronization word can be chosen from a variety of schemes: it can correspond to a sequence of pulses, a Pseudo-Noise (PN) sequence [20] or the 802.11 preamble [21]. In all these cases, the synchronization word has a certain duration T_{Tx}.

During the transmission of this word, a node is unable to receive. This constraint is due to limitations on the Radio Frequency (RF) part of transceivers.

After the message has propagated and been received by node i, some processing time is required to correctly declare that a synchronization message has been received. This results in a decoding delay T_{dec}.

Altogether, four delays need to be taken into account to model the synchronization strategy to a wireless network:

- $T_0^{(i,j)}$: Propagation delay - time taken for a burst to propagate from the emitting to the receiving node. This time is proportional to the distance between two nodes.
- T_{Tx}: Transmitting delay - length of the synchronization word. A node cannot receive during this time.
- T_{dec}: Decoding delay - time taken by the receiver to properly identify the emissions of a synchronization word. This time needs to be overestimated to account for the slowest receiver.

- T_{refr}: Refractory delay - time necessary after transmitting to maintain stability.

In this paper, all propagation delays are considered negligible in order to focus our analysis on the effects of the transmission time and the decoding delay on the original scheme. This assumption is valid when considering Wireless LAN settings in an ad hoc scenario. Typically the maximum operation range is $50\,\text{m}$, which limits the propagation delay to $T_0^{[\text{max}]} = \frac{50\,\text{m}}{c} \approx 0.17\,\mu\text{s}$. In comparison, the preamble of an 802.11 frame, which can be used as the synchronization word, has a duration of $T_{\text{Tx}} = 8\,\mu\text{s}$ [21]. Noise at the receiver and channel gain are also neglected to emphasize the effect of delays on the original scheme.

For simplicity, transmission and decoding delays T_{Tx} and T_{dec} are the same for all nodes. To assume that the decoding delay is the same for all nodes, slow receivers need to be accounted for. Therefore, in practice, this delay should be overestimated, so that all nodes increment their phases simultaneously upon proper reception of a synchronization word.

To illustrate the impact of these delays, we choose the synchronization word to be a PN sequence. When node j fires, it enters a transmit state: a synchronization word $x(t)$ passes through a shaping filter and starts being emitted. It then propagates through a channel $h(t)$ before being received by node i, which collects the incoming signal through a matched filter and samples it [20]. As $x(t)$ is the same for all nodes, node i detects it by correlating the received signal with the known message. The output of the correlation detector is given by [20]:

$$\Lambda_i(t) = x(-t) * y_i(t) \tag{7}$$

where $y_i(t)$ is the incoming signal at node i and $*$ denotes the convolution operator.

From the system model described previously, Fig. 3 plots the output of the correlation detector $\Lambda_i(t)$ and the corresponding phase function of node i during this period.

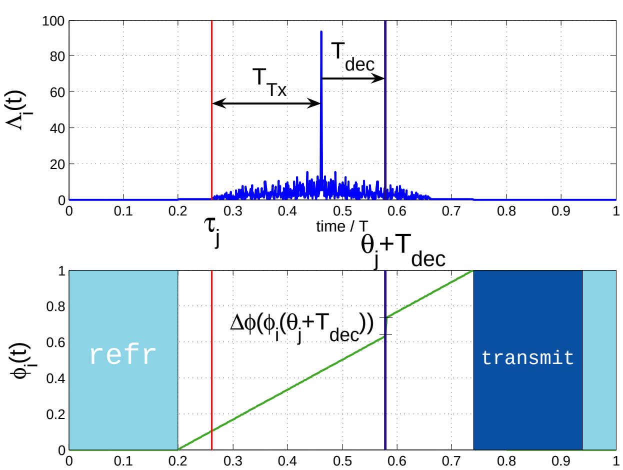

Fig. 3. Example of the output of the correlator when considering noise

On Fig. 3 a synchronization sequence of duration $T_{\text{Tx}} = 0.2 \cdot T$ is transmitted by node j at $t = \tau_j$, and is collected by the receiver. The output of the correlator produces a large peak that occurs at exactly $\theta_j = \tau_j + T_{\text{Tx}}$ [20]. In order to properly identify this peak, node i increments its phase $T_{\text{dec}} = 0.12 \cdot T$ after the peak, i.e. at $\theta_j + T_{\text{dec}}$.

These delays are the most significant difference from the Mirollo and Strogatz model, which assumes no propagation delay, an infinitely short transmission time and no decoding delay [1]. The total delay is defined by:

$$T_{\text{del}} = T_{\text{Tx}} + T_{\text{dec}} \tag{8}$$

When several nodes transmit, if the PN sequence has low auto-correlation properties such as a Gold sequence [22], then distinct peaks appear at the output of the correlator Λ_i exactly T_{del} after a node has fired [20]. Thus several transmissions are distinguishable and several phase increments occur. If nodes fire and transmit synchronously, peaks superimpose constructively.

The total delay T_{del} represents the inherent time difference between the beginning of the transmission of a synchronization burst and its successful reception. From the theory of coupled oscillators, it is known that delays impact heavily on the synchronization process and its stability [23]. For pulse-coupled oscillators, it has been shown that the system becomes unstable in the presence of delays [24], but this model does not account for the transmission and refractory periods, where no coupling is possible. In our model, as coupling is only possible when nodes are in a listening state, stability issues are prevented by adjusting T_{refr}, and proper choice for T_{refr} is done through simulations.

In any case, due to T_{del} and to the fact that during transmission it is not possible to receive, a *deafness* of duration T_{del} appears in which nodes cannot listen to the network. Within this deafness, no mutual coupling between nodes can occur, which implies that the attainable synchronization accuracy is lower bounded by T_{del}. For transmission techniques where the time for one symbol block, T_{Tx}, cannot be assimilated as a pulse, such an accuracy is clearly unacceptable. Therefore, there is a need to modify the synchronization strategy.

C. Compensating Delays

In order to regain high accuracy, we propose to combat transmission delays by modifying the intrinsic behavior of a node: after firing, a node *delays* its transmission of the synchronization word. This approach is similar to the one observed in the experiments of fireflies: in response C of Fig. 1, the advance in flashing is not effective immediately upon reception of a signal, but occurs in the following period. The waiting delay is chosen to be:

$$T_{\text{wait}} = T - T_{\text{del}} = T - (T_{\text{Tx}} + T_{\text{dec}}) \tag{9}$$

where T denotes the synchronization period. With this approach, receivers increment their phases exactly T seconds after a transmitter fired.

This scheme modifies the natural oscillatory period of a node, which is now equal to $2 \cdot T$. Nodes are coupled only if they can hear each other during T_{Rx}, which is the time during which the phase function will linearly increments over time. This time is reduced by the waiting, transmitting, and refractory delays, and is now equal to:

$$T_{Rx} = 2 \cdot T - (T_{wait} + T_{Tx} + T_{refr}) = T + T_{dec} - T_{refr} \quad (10)$$

To summarize the modified behavior of a node, Fig. 4 represents the four successive states of a node: `wait`, `transmit`, `refr` and `listen` when $T_{wait} = 0.75 \cdot T$, $T_{Tx} = 0.2 \cdot T$, $T_{dec} = 0.05 \cdot T$, $T_{refr} = 0.2 \cdot T$ and $T_{Rx} = 0.85 \cdot T$. A node is represented as a marker that circles around the phase diagram linearly over time and counterclockwise. Using this diagram, N nodes can be represented on the same circle, which helps analyzing the dynamic evolution of the system. One full rotation of a marker corresponds to a period $2 \cdot T$.

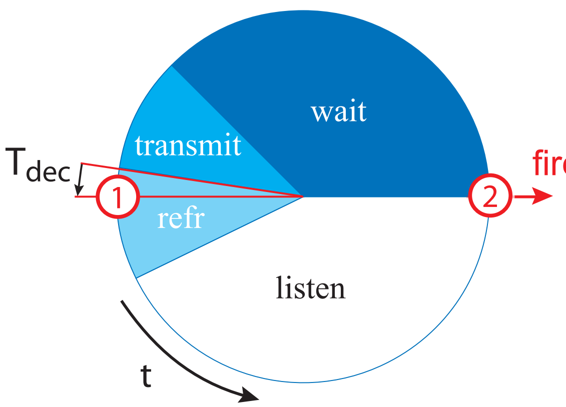

Fig. 4. Phase diagram of a system of N nodes following the novel synchronization strategy. Two groups of oscillators form, spaced exactly T apart.

For a system of N oscillators, all firings instants are initially randomly distributed over a period of $2 \cdot T$, i.e. all markers are randomly uniformly distributed around the circular state machine representation. Each oscillator follows the same rules of waiting before transmitting. When a message is successfully received during `listen`, the marker representing this node abruptly shifts its position towards the diametrically opposed state on the representation of Fig. 4. Over time the oscillators split into two groups diametrically opposed on the state machine representation, each group firing T seconds apart and helping each other to synchronize. Therefore T is still used as the reference synchronization period.

The formation of two groups is a necessary requirement for maintaining high accuracy, because nodes that transmit almost simultaneously cannot hear each other (deafness while transmitting). Therefore each group helps the other to synchronize by transmitting T seconds after the other.

With this new transmitting strategy the accuracy of synchronization is no longer limited by T_{del}. Successful synchronization is therefore declared when firing instants are spread over a time interval that is equal or smaller than the maximum propagation delay.

D. Simulation Results

Deriving a thorough mathematical demonstration that synchrony is reached when each node follows the simple rules of waiting before transmitting is a task of formidable difficulty, as proving synchrony when no delays are present is already difficult. The behavior of the system lies within the field of nonlinear dynamics, and a complete description and analysis does not seem easily reachable. Therefore we rely on simulation results to evaluate our synchronization scheme.

To verify the validity of the synchronization scheme, Monte Carlo simulations are carried out. Fig. 5 plots the synchronization rate for several values of T_{Tx}, T_{refr} and α. For simulations, each period T is decomposed into 1500 steps, and at each step, state and interactions of each node are evaluated. The decoding delay is fixed to $T_{dec} = 0.1 \cdot T$. Nodes are able to perfectly distinguish each transmitted synchronization word, e.g. by using a long Gold sequence as the synchronization word. The initial conditions correspond to the case where all nodes have randomly distributed state variable, i.e. each node is assumed to be active starting with a random phase $\phi_i(0)$. Successful synchronization is declared if two groups of oscillators firing T seconds apart form, and the synchrony rate is defined as the number of successful synchronizations after $50 \cdot T$ over the number of realizations of initial conditions, which is set to 1000. Initial conditions correspond to the worst case scenario where initially all state variables are randomly distributed around the phase diagram of Fig. 4.

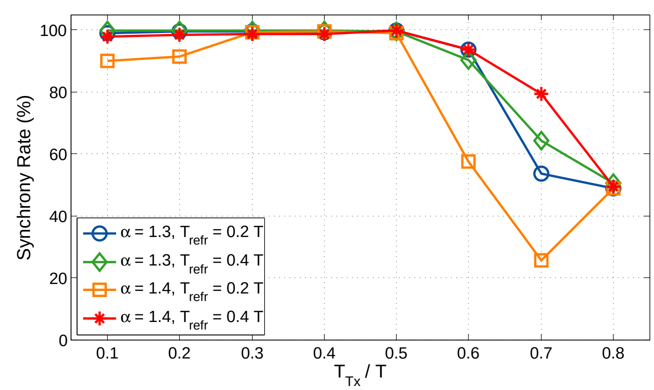

Fig. 5. Synchronization rate when T_{Tx} varies

For $T_{refr} = 0.2 \cdot T$, $\alpha = 1.4$ (high coupling), and low values of T_{Tx}, the system is unstable and synchrony is not always reached. In this case, clusters of oscillators form and oscillate more rapidly (phase-locking mechanism). This phenomenon tends to disappear when the transmitting time increases, but it is never completely resolved and the synchrony rate is never higher than 90%.

When increasing the refractory time T_{refr} to $0.4 \cdot T$, synchronization is always obtained for a large range of T_{Tx}. Thus, a relatively long refractory time is preferable. For $T_{Tx} > 0.6 \cdot T$, the synchrony rate becomes lower than 90% and drops very rapidly. This can be explained by the proportion of the listening time, which becomes small compared to the transmitting time. This makes it difficult for nodes to hear another and reach a consensus.

Fig. 6 plots the mean time taken by a system of 30 oscillators to synchronize. The duration of a time slot T is used as reference.

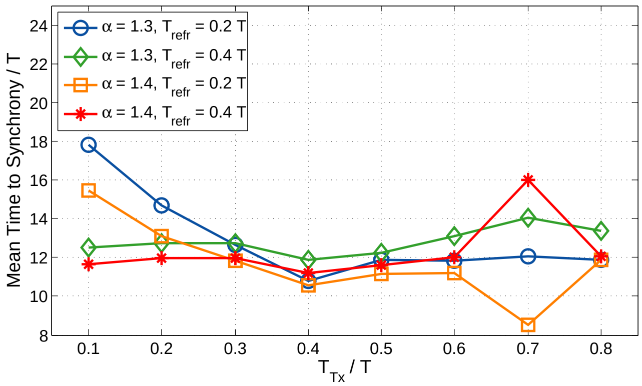

Fig. 6. Mean time to synchrony when T_{Tx} varies

For $T_{\text{refr}} = 0.2 \cdot T$, the mean time to synchrony $\bar{T}$ is lower than for $T_{\text{refr}} = 0.4 \cdot T$. However synchrony rates are lower and it is preferable to ensure that synchrony will be reached although it may take one or two periods more in average.

For $T_{\text{refr}} = 0.4 \cdot T$, $\bar{T}$ is lower when $\alpha = 1.4$ than with $\alpha = 1.2$. This complies with the original Mirollo and Strogatz model where the mean time to synchrony is inversely proportional to the coupling factor α.

V. Conclusion

Fireflies provide an amazing spectable with their ability to synchronize using simple rules: each node maintains an internal clock dictating when to emit, and in return, this clock is adjusted when receiving. These synchronization rules are particularly simple and well suited for a deployment in ad hoc networks. However they are not directly applicable when accounting for transmission delays and the fact that a node cannot receive and transmit simultaneously.

To regain a level of accuracy that is upper bounded by the propagation delay, the intrinsic behavior of nodes was modified to compensate for transmission and decoding delay. Thanks to this modification, communication through pulses is no longer required, and accurate synchrony of oscillators is possible. The simplicity and generality of the synchronization scheme makes its implementation very appealing. If all nodes cooperate, synchrony can be reached within 15 periods. Once nodes have agreed on a common time scale, they are then able to use the full time slot to communicate in a synchronous manner.

While the rules of the novel synchronization strategy are simple, the waiting time imposes additional delays, raising the constraints to achieving convergence and stability. In a not fully-meshed multi-hop network, however, the situation is more complicated. Due to the fact that two groups are established, formations might occur where one node is surrounded by nodes which are all in the same group. This may result in a “deafness effect”, where a local group of nodes all transmit at similar time instants, which implies that these nodes cannot hear each other. While for a fully-meshed network the probability that all local nodes are within one group tends to zero, the deafness effect causes severe problems for meshed networks, and is a suitable topic for further research.

References

[1] R. Mirollo and S. Strogatz, “Synchronization of pulse-coupled biological oscillators,” *SIAM J. Appl. Math*, vol. 50, no. 6, pp. 1645–1662, Dec. 1990.

[2] D. Attenborough, “BBC Trials of Life, “Talking to Strangers”,” 1990.

[3] D. Otte and J. Smiley, “Synchrony in texas fireflies with a consideration of male interaction models,” *Biology of Behaviour*, vol. 2, pp. 143–158, 1977.

[4] C. Peskin, *Mathematical Aspects of Heart Physiology*. New York: Courant Institute of Mathematical Sciences, 1975.

[5] J. Hopfield and A. Herz, “Rapid local synchronization of action potentials: Toward computation with coupled integrate-and-fire neurons,” *Proc. National Academy of Sciences USA 92*, vol. 92, pp. 6655–6662, July 1995.

[6] S. Strogatz and I. Stewart, “Coupled oscillators and biological synchronization,” *Scientific American*, vol. 269, no. 6, pp. 68–74, Dec. 1993.

[7] A. Winfree, “Biological rhythms and the behavior of populations of coupled oscillators,” *J. Theor. Biol.*, vol. 16, no. 1, pp. 15 – 42, July 1967.

[8] Y.-W. Hong and A. Scaglione, “Time synchronization and reach-back communications with pulse-coupled oscillators for UWB wireless ad hoc networks,” in *Proc. IEEE Conference on Ultra Wideband Systems and Technologies 2003*, Nov. 2003, pp. 190–194.

[9] G. Werner-Allen, G. Tewari, A. Patel, M. Welsh, and R. Nagpal, “Firefly-inspired sensor network synchronicity with realistic radio effects,” in *Proc. SenSys ’05*, Nov. 2005.

[10] A. Tyrrell, G. Auer, and C. Bettstetter, “Firefly synchronization in ad hoc networks,” in *Proc. MiNEMA Workshop 2006*, Feb. 2006.

[11] S. Strogatz, *Sync: The Emerging Science of Spontaneous Order*, 1st ed. Hyperion, 2003.

[12] S. Camazine, J.-L. Deneubourg, N. R. Franks, J. Sneyd, G. Theraulaz, and E. Bonabeau, *Self-Organization in Biological Systems*. Princeton, 2001.

[13] C. Richmond, “Fireflies flashing in unison,” *Science*, vol. 71, pp. 537–538, 1930.

[14] J. Buck, E. Buck, J. Case, and F. Hanson, “Control of flashing in fireflies. V. Pacemaker synchronization in *pteroptyx cribellata*,” *Journal of Comparative Physiology A*, vol. 144, pp. 630–633, 1981.

[15] G. M. R. Ávila, “Synchronization phenomena in light-controlled oscillators,” Ph.D. dissertation, Université Libre de Bruxelles, 2004.

[16] S. R. Campbell, D. L. Wang, and C. Jayaprakash, “Synchrony and desynchrony in integrate-and-fire oscillators,” *Neural Computation*, vol. 11, no. 7, pp. 1595–1619, 1999.

[17] B. Ermentrout, “An adaptive model for synchrony in the firefly *pteroptyx malaccae*,” *Journal of Mathematical Biology*, vol. 29, no. 6, pp. 571–585, 1991.

[18] A. Hu and S. Servetto, “On the scalability of cooperative time synchronization in pulse-connected networks,” *IEEE Transactions on Information Theory*, vol. 52, pp. 2725–2748, June 2006.

[19] U. Ernst, K. Pawelzik, and T. Geisel, “Synchronization induced by temporal delays in pulse-coupled oscillators,” *Phys. Rev. Lett.*, vol. 74, no. 9, pp. 1570–1573, Feb. 1995.

[20] J. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.

[21] *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, IEEE Std. 802.11a, 1999.

[22] R. Gold, “Optimal binary sequences for spread spectrum multiplexing,” *Information Theory, IEEE Transactions on*, vol. 13, no. 4, pp. 619–621, Oct. 1967.

[23] M. Yeung and S. Strogatz, “Time delay in the Kuramoto model of coupled oscillators,” *Phys. Rev. Lett.*, vol. 82, no. 3, pp. 648 – 651, 1999.

[24] M. Timme, F. Wolf, and T. Geisel, “Prevalence of unstable attractors in networks of pulse-coupled oscillators,” *Phys. Rev. Lett.*, vol. 89, no. 15, Oct. 2002.