Cooperative Multicast with Low-Cost Radios

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***Abstract*—A simple cooperative diversity technique is applied to point-to-multipoint communication in wireless medium suffering from Rayleigh block fading. Such a concept is especially interesting if low-cost radios, which have no advanced capabilities for fading mitigation, are used. This paper analyzes the communication reliability in terms of outage and delivery ratio, and compares cooperative and non-cooperative schemes. Results from a simple setup show that already few preselected relays can lead to performance gains. A potential application field is link-layer broadcast in a network of energy-constrained embedded devices.**

***Index Terms*—Cooperative multicast, cooperative broadcast, cooperative diversity, relay, energy constraints, Rayleigh fading.**

I. INTRODUCTION

The use of cooperative diversity in wireless communications is considered to be an efficient means to mitigate negative effects of multipath propagation (see, e.g., [1]–[5]). While most research in this area has been on *point-to-point* communication, this paper addresses the use of cooperative diversity for *point-to-multipoint* communication. The setting is as follows: A sender intends to deliver a message to multiple adjacent nodes in a multipath propagation environment. Due to small-scale fading, not all destination nodes may successfully decode the message. Hence, one or more of the destinations which actually received the message act as relays, re-transmitting the message on the wireless link. Such application of cooperative diversity in a multicast scenario is attractive, as nodes that are anyway destinations act as relays. This approach is called *cooperative multicast* in the following.

We are interested in the use of cooperative multicast in networks with low-cost radios as found, for example, in tiny sensors or other embedded devices. Such radios do not have advanced capabilities of fading mitigation (e.g., MIMO) due to severe limitations on the hardware size, cost, and complexity. Cheap radios are usually characterized by strict channel orthogonality in time, cheap antennas, simple protocols, no energy accumulation, and no or simple power adjustment mechanisms. Furthermore, it is assumed that such devices must be especially energy-efficient; the achieved data rates and delays are assumed to be much less important.

This paper analyzes the reliability of cooperative multicast in terms of various performance metrics, such as the percentage of nodes finally receiving the message correctly. Under given energy constraints, the performance of cooperative multicast is compared to non-cooperative multicast with and without time diversity. To keep the mathematical analysis simple, a scenario with nodes positioned at equal distances on a radial ray from the sender is used.

The paper is organized as follows. Section II covers related work. Section III introduces the used models for the radio channel and the network. Section IV explains three multicast techniques (direct multicast, time-diversity multicast, and cooperative multicast) and derives various performance measures for these techniques. Section V presents and discusses the performance results. Finally, Section VI concludes the paper.

II. RELATED WORK

Numerous publications address energy-efficient multicast in non-fading environments. In [6] and [7] authors show that the problem of optimal transmit power control and multicast tree construction in a network is NP-complete; they propose heuristic algorithms to achieve energy efficiency with lower computational complexity.

The paper [8] discusses cooperative diversity in a multihop network. It is assumed that receivers are capable to accumulate energy of multiple simultaneous transmissions. When accumulated energy exceeds a certain threshold, information can be decoded correctly and broadcasted further. With this assumption the problem of optimal power allocation for data broadcast remains NP-complete; heuristic algorithms are proposed and analyzed in [9]–[11]. The analysis of multicast techniques in these papers is however made in non-fading environments with instantaneous knowledge of the link qualities between nodes, which in presence of multi-path fading would require significant coordination efforts. The paper [12] proposes a network-wide flooding approach in multihop networks using cooperative diversity.

The paper [13] shows that cooperation with feedback provides substantial gains for simple low-cost radios with strictly orthogonal transmissions. The authors of [14] and [15] study cooperative multicast to deliver lossy data with differentiated quality under delay constraints in Rayleigh fading networks.

III. MODELING ASSUMPTIONS

A. Channel with Rayleigh Block Fading

We assume a radio channel with distance-dependent path loss, multipath fading, and additive white Gaussian noise. The signal-to-noise ratio (SNR) at the sender s is $\mathrm{SNR}_s = P_{\mathrm{Tx}}/P_n$ with the transmit power P_{Tx} and the noise power P_n. A node i is located at distance d_i from this sender (and this distance is normalized to one meter). The signal power of s is attenuated

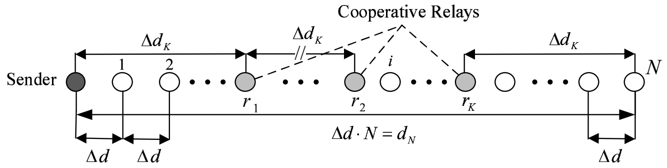

Fig. 1. One-dimensional topology with equally spaced nodes and relays.

over this distance and suffers from multipath fading. The SNR at node i can be expressed as

$$\mathrm{SNR}_{si} = d_i^{-\alpha}\, h^2\, \mathrm{SNR}_s, \tag{1}$$

where the constant α is the pathloss exponent, and the random variable h models the fading. The variable h is Rayleigh distributed with $\sigma^2 = 1/2$. Thus, the instantaneously received SNR is exponentially distributed with mean value $d_i^{-\alpha}\,\mathrm{SNR}_s$. Similar channel models are used in other studies of cooperative diversity in fading environments (see [2], [3]). We assume slow fading modeled as block fading, where the receiver SNR remains constant for one packet transmission.

B. One-Dimensional Topology with Equally Spaced Nodes

We use a one-dimensional setup as shown in Figure 1. Networks of this kind can be found for example in transportation, production, and control systems. The most left node is the sender s. There are N destination nodes placed in the form of a ray from the sender. The node with the largest distance from the sender is located at distance d_N, which is set to the expected range of the sender ($h = 1$, this range is called path loss range). This yields

$$\mathrm{SNR}_{\min} = d_N^{-\alpha}\,\mathrm{SNR}_s \;\Leftrightarrow\; d_N = \sqrt[\alpha]{\frac{\mathrm{SNR}_s}{\mathrm{SNR}_{\min}}}\,. \tag{2}$$

The distance between two consecutive nodes is $\Delta d = d_N/N$. The distance d_i from the sender to node i is $d_i = i\Delta d = i\,d_N/N$, where $i \in \{1,..,N\}$ is the counting index starting with $i = 1$ for the node closest to the sender.

C. Preselected, Equally Spaced Relays

Let K denote the number of nodes acting as relays. The problem of selecting relays that optimize multicast performance for a given number of nodes N and relays K is of high interest but out of the scope of this paper. Similar problems in non-fading environments are NP-complete (see [6]–[8]). Instead, we study the case of predetermined relays (see Figure 1). The relays are chosen from the set of nodes in a way that the distance between two adjacent relays is always

$$\Delta d_K = \Delta d \left\lfloor \frac{N}{K+1} \right\rfloor, \text{ for } 1 \le K \le N-1. \tag{3}$$

Such placement is easy to implement and intuitively beneficial, as relays can be activated in a cascade-like manner to deliver information to remote nodes via multiple hops.

IV. MULTICAST TECHNIQUES AND PERFORMANCE METRICS

A. Multicast Techniques

We now investigate the reliability performance of cooperative multicast and relate it to non-cooperative multicast. We compare the following three techniques:

- Using *direct multicast* (DM), the sender transmits a message once with full power P_{Tx}.
- Using *direct multicast with time diversity* (DMT), the sender transmits a message twice, each transmission with half power $P_{\mathrm{Tx}}/2$ (diversity order two).
- Using *cooperative multicast* (CM) with K relays, the sender transmits once with power $P_{\mathrm{Tx}}/(1+K)$, subsequently, each of the relays transmits with $P_{\mathrm{Tx}}/(1+K)$.

The scaling of the transmit power is done to obtain a comparison that is fair in terms of energy resources; i.e., the total used energy is the same in each of the techniques and independent of the number of relays. This energy-fairness is our major concern here. Each node transmits with the same rate. Hence, the comparison is unfair in terms of the used time resources. This restricts the comparison to scenarios where energy not data rate is the limiting resource. Examples include networks of embedded devices that have to be energy-efficient .

Furthermore, we assume that each message experiences independent fading. In contrast to the models in [8]–[11], we assume that accumulation of energy is impossible (simple radios). No combining schemes are used. Instead, received messages are dropped if the receiver SNR is below the threshold $\mathrm{SNR}_{\min}$. Such approach of selective combining is reasonable in a slow fading environment and is shown to perform just slightly worse than maximum ratio combining on the symbol level [16]. A relay retransmits only once if it correctly receives the message from the source or other relays. We assume that cooperative relays are selected proactively (before the source starts to transmit) and their number K is known to the source and to the relays.

B. Definition of Performance Measures

The performance analysis is based on the occurrence of *link outages* between the sender and destination nodes. A link outage to a destination occurs whenever the SNR falls below a certain sensitivity threshold $\mathrm{SNR}_{\min}$. This means that the message cannot be decoded correctly by the destination node. Given the modeling assumptions described above, the outage probability for a transmission from the sender s to node i is

$$p_{si} \;=\; \mathrm{P}\big[\mathrm{SNR}_{si} < \mathrm{SNR}_{\min}\big] = 1 - \exp\left(-\frac{d_i^{\alpha}\,\mathrm{SNR}_{\min}}{\mathrm{SNR}_s}\right). \tag{4}$$

For a given message transmission from the sender, we can determine the *delivery ratio* R of this message in the network; it is the fraction of nodes that receive the message correctly. It is a commonly used reliability metric in group communications. Since message delivery is a probabilistic event in a fading channel, the ratio R is a random variable.

We are interested in the following performance measures:

- The maximum outage probability $p_{\max} = \max_i p_{si}$ is the outage probability of the node that has the highest outage probability among all nodes. It is considered because some nodes might experience very high individual outage probabilities even if the group delivery ratio is acceptable. It makes a statement about the minimal reliability level.
- The expected delivery ratio $E[R]$ is the expected value of R, i.e., the expected fraction of nodes that receive a message correctly.
- Finally, $\mathrm{P}[R \geq \gamma]$ is the probability that the delivery ratio is at least γ, i.e., a fraction γ of all nodes will receive a message sent by the source. The probability of full delivery is $\mathrm{P}[R = 1]$.

C. Derivation of Performance Measures

1) Direct Multicast: A message is delivered successfully to a node if the SNR of the sender's message is above the threshold. Applying the assumptions of Sections III-A and B in (4), the outage probability of a direct transmission from the source s to node i simplifies to

$$p_{si} = p_i = 1 - \exp\left(-\left(\frac{i}{N}\right)^{\alpha}\right). \tag{5}$$

The maximum outage probability occurs at node N. For direct multicast it yields

$$p_{\max} = \max_{i \in \{1,..,N\}} p_i = e^{-1} = 63.21\,\% \,. \tag{6}$$

The expected delivery ratio is

$$E[R] = \frac{1}{N}\sum_{i=1}^{N}(1 - p_i) = \frac{1}{N}\sum_{i=1}^{N}\exp\left(-\left(\frac{i}{N}\right)^{\alpha}\right), \tag{7}$$

and the probability of full direct delivery is

$$\mathrm{P}[R = 1] = \prod_{i=1}^{N}(1 - p_i) = \prod_{i=1}^{N}\exp\left(-\left(\frac{i}{N}\right)^{\alpha}\right). \tag{8}$$

2) Direct Multicast with Time Diversity: A message is delivered successfully to a node if at least one of the two messages is received with an SNR above the threshold. The outage probability is given by the probability that both messages fail. Considering the reduced transmit power (i.e., replacing SNR_s with $\mathrm{SNR}_s/2$), the overall outage probability to node i is

$$p_i = \left[1 - \exp\left(-2\left(\frac{i}{N}\right)^{\alpha}\right)\right]^2. \tag{9}$$

The maximum outage probability occurs at node N. The maximum outage probability is then $p_{\max} = 74.76\,\%$. The expected delivery ratio and the probability of full delivery can be computed using (9) in (7) and (8), respectively.

3) Cooperative Multicast: Using cooperative multicast, a message will be delivered successfully to a non-relay node i if it is received directly from the sender or/and by at least one of the K relays. A relay retransmits a message only once if it receives it itself from the sender or other relays correctly at a previous time instant. For the calculation of the average outage probability over all nodes we must calculate the individual outage probabilities of each possible path and combine them. Furthermore, we must account for the reduced transmit power $P_{\mathrm{Tx}}/(1+K)$ of each transmission.

The outage probability at node i for a direct transmission from the source s is

$$p_{si} = 1 - \exp\left(-(K+1)\left(\frac{i}{N}\right)^{\alpha}\right). \tag{10}$$

The outage probability of a transmission from relay r to node i is

$$p_{ri} = 1 - \exp\left(-(K+1)\left(\frac{|r-i|}{N}\right)^{\alpha}\right). \tag{11}$$

A message can be delivered to a given non-relay node i via

$$M = \sum_{\kappa=0}^{K}\kappa!\binom{K}{\kappa} \tag{12}$$

different paths, including the direct sender-destination path. These paths are indexed by the integer $j \in \{1, \ldots, M\}$ in the following. The event that a message is delivered successfully via path j is denoted as $\mathcal{S}_j$. The outage probability at a non-relay node i can be expressed via the probability of the union of events $\mathcal{S}_j$ or the intersection of complementary events $\overline{\mathcal{S}}_j$. For a given set of relay nodes, we have

$$p_i = 1 - \mathrm{P}\left[\bigcup_{j=1}^{M}\mathcal{S}_j\right] = \mathrm{P}\left[\bigcap_{j=1}^{M}\overline{\mathcal{S}}_j\right]. \tag{13}$$

If only one relay r is used ($K = 1$), the overall outage probability at node i can be calculated by

$$p_i = \begin{cases} p_{si}\left(1 - (1 - p_{sr})(1 - p_{ri})\right) & \text{for } i \neq r \\ p_{si} & \text{for } i = r \end{cases}, \tag{14}$$

with p_{si} and p_{ri} given by (10) and (11), $K = 1$, and $r = \lfloor N/2 \rfloor$. The sender-relay outage probability p_{sr} is given by (10) with $i = r$. The resulting maximum outage probability in this case is

$$p_{\max} = 0.8646 - 0.8646\, e^{-4 \cdot 2^{-\alpha}} = 34.02\,\% \tag{15}$$

with $\alpha = 3$. The expected delivery ratio can be calculated as above. The probability of full delivery is

$$\mathrm{P}[R = 1] = (1 - p_{sr})\prod_{i=1}^{N-1}(1 - p_{si}\,p_{ri}). \tag{16}$$

V. PERFORMANCE RESULTS

Let us now compare the performance of the three multicast techniques. We use the analytical expressions derived in the previous section for DM, DMT, and CM with one relay and backup these results by simulation. Results for CM with more than one relay have been obtained only by simulations due to the dependencies of the events $\mathcal{S}_j$. In the following figures, lines without markers correspond to analytical results, and lines with markers to simulation results. The path loss exponent is always $\alpha = 3$.

We distinguish between two scenarios: In the first scenario, all nodes are destinations (full multicast range). In the second scenario, only a subset of the nodes — the ones closer to the sender — are destinations (reduced multicast range).

A. Full Multicast Range

Table I compares DM, DMT, and CM with $K = 1 \dots 5$ relays for $N = 50$ nodes based on the three reliability measures. Time diversity only slightly improves the expected delivery ratio. Due to the halved transmission power it becomes difficult to reach distant nodes, thus the maximum outage probability increases. Cooperative multicast leads to a performance improvement. All measures become better with an increasing number of relays. For the given scenario, just a few relays are needed to achieve a high expected delivery ratio, small maximum outage probability, and probability of full delivery probability close to $100\,\%$.

TABLE I
RELIABILITY OF DIRECT MULTICAST (DM), DM WITH TIME DIVERSITY (DMT), AND COOPERATIVE MULTICAST (CM), $N = 50$ NODES.

			CM, number of relays K				
Values in %	DM	DMT	1	2	3	4	5
Delivery ratio	80.1	81.3	90.7	96.5	98.8	99.6	99.8
Max. outage	63.2	74.8	34.0	14.4	5.0	1.5	0.5
Full delivery	0.0	0.0	16.9	60.5	85.5	95.3	98.5

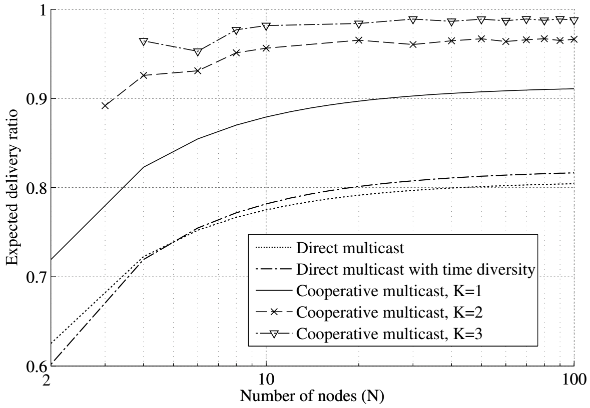

Fig. 2. Expected delivery ratio $\mathrm{E}[R]$ over the number of nodes.

Figure 2 shows the expected delivery ratio $\mathrm{E}[R]$ as a function of the number of nodes N from 2 to 100. Two observations can be made: First, for arbitrary N, the delivery ratio improves with an increasing number of relays. Second, the delivery ratio grows with increasing N for all multicast strategies. This behavior can be explained by the fact that the outage probability is a non-linear function of the sender-to-destination distance.

Figure 3 depicts the probability $\mathrm{P}[R \geq \gamma]$ that a desired delivery ratio of at least γ can be achieved in a network with $N = 50$ nodes. On the one hand, non-cooperative multicast outperforms cooperative multicast if the desired delivery ratio γ is low. It outperforms one relay for $\gamma \leq 80\,\%$ and more relays below $75\,\%$. On the other hand, if a high delivery ratio is needed, cooperative multicast shows a much better performance. For instance, multicast with three relays achieves a delivery ratio of $\gamma = 97\,\%$ with the probability of about $95\,\%$.

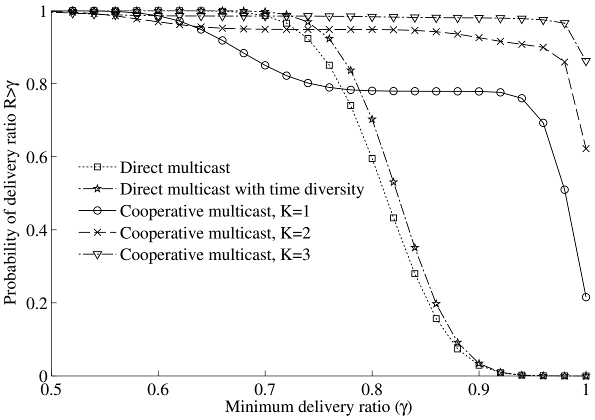

Fig. 3. Probability $\mathrm{P}[R \geq \gamma]$ that a desired minimum delivery ratio γ can be achieved, $N = 50$.

B. Reduced Multicast Range

Finally, we study the performance when the multicast range is lower than the pathloss range d_N. It is no longer required that all N nodes obtain the sender's message, but only nodes with indexes $i \leq N_s$ are considered as destinations. Full delivery is achieved if all these nodes receive the message. Relays are selected among these N_s nodes according to (3) by replacing N by N_s.

The following figures show performance curves as a function of N_s for $N = 100$ nodes. All figures illustrate in which scenarios cooperation can lead to gains.

Figures 4 and 5 show that CM outperforms DM and DMT in terms of delivery ratio and maximum outage if more than about $20\,\%$ of the nodes should be reached. CM with one relay performs very similar to DMT up to $N_s/N \approx 45\,\%$. This indicates that in this range CM mainly helps to overcome small-scale fading. If the percentage of nodes-to-reach is higher (about $N_s/N > 45\,\%$), CM outperforms time diversity. This is because relaying also leads to a multihop gain that contributes beneficially to the distance-dependent pathloss. Due to cascade effects additional relays help to improve end-to-end pathloss for distant nodes and enhance the multicast reliability when the multicast range increases.

Figure 6 shows the probability of full delivery as a function of the multicast range. The benefits of time diversity and cooperation can be seen already for $N_s/N > 10\,\%$. This can be explained by the multiplicative nature of the metric, as shown in (8), where even slight improvements of individual outage probabilities result in a noticeable probability increase of the full delivery.

VI. CONCLUSIONS AND FURTHER WORK

The goal of this paper was to analyze whether the use cooperative relaying can improve the reliability of point-to-multipoint communication on a wireless link that suffers from multipath fading. We assumed to have cheap and energy-constrained radios, as used in some types of networked

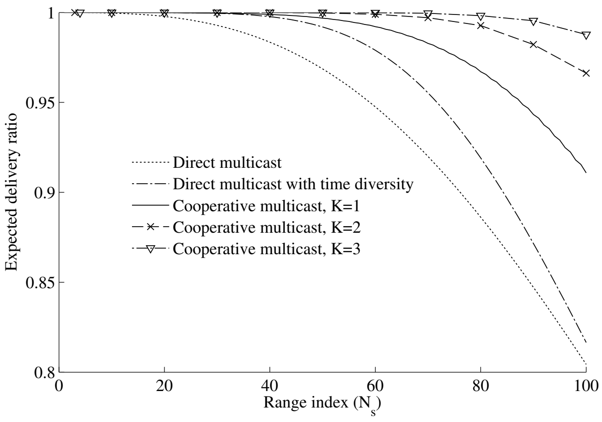

Fig. 4. Expected delivery ratio $\mathrm{E}[R]$ over the multicast range N_s; $N = 100$.

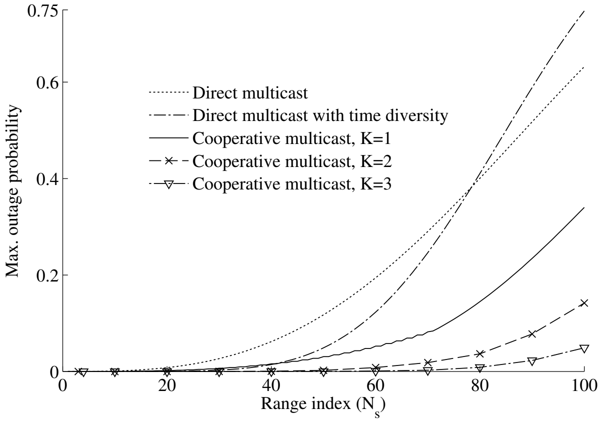

Fig. 5. Maximum outage probability over the multicast range N_s; $N = 100$.

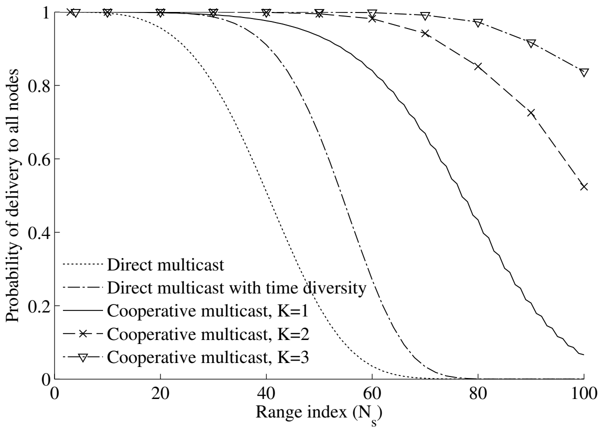

Fig. 6. Probability of full delivery $\mathrm{P}[R = 1]$ over N_s; $N = 100$.

embedded systems. The relay selection and power allocation were performed in a suboptimal but practically feasible and straightforward manner. The analysis showed that significant reliability improvements can be achieved with already very few cooperative nodes. The realization of such predetermined cooperative multicast techniques does not require much signaling overhead. The investigated scenario may occur in applications where the network is embedded into objects, like walls or trains. Furthermore, despite the simple scenario, it can be expected that the main tendencies are also visible in more complicated setups (e.g., nodes distributed in two dimensions). The analysis of such setups is subject to our current research.

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