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# Impact of Random Mobility on the Inhomogeneity of Spatial Distributions

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***Abstract*—Simulation results of wireless networks heavily depend on the spatial distribution of its nodes. Even though the initial distribution may match the expectations of the researcher, its properties may get lost due to the applied mobility model after a few seconds. This paper analyzes the effects of three well-known mobility models on the inhomogeneity of the spatial distribution. Furthermore, the effects of penetrable borders of the simulation area on the node distribution are analyzed. Due to the encountered deviations, we propose and analyze the inhomogeneous random waypoint (IRWP) model. It maintains a desired inhomogeneity level in the long run and generates random clusters with respect to shape, size, position, and member nodes.**

***Index Terms*—Wireless networks, simulation, mobility model, inhomogeneity**

## I. Introduction and Motivation

The majority of current research in wireless multihop networking assumes that the nodes are uniformly randomly placed over the system area. Such a homogeneous distribution is convenient in analytical studies as it enables us to calculate the coverage, connectivity, capacity, and other performance parameters. It is also convenient in network simulations as most simulation tools offer integrated uniform random node placement. In real networks, however, the nodes are in general not uniformly distributed but concentrate around hot spots; they form clusters and leave parts of the system area unoccupied. This is especially true if we consider sparsely-connected networks, such as delay-tolerant community networks and vehicular ad hoc networks.

The discrepancy between research modeling and reality has motivated some researchers to propose models for *inhomogeneous* node distribution and use them in their simulations (see [1]–[4]). For instance, the authors' work [4] proposes a method to generate random inhomogeneous distributions based on a neighbor-dependent thinning of a random homogeneous distribution. The work does not only define a model for inhomogeneous node placement but also derives some important stochastic properties of the resulting distribution, such as the probability density of the nearest neighbor distance. The proposed method is flexible in a way that two parameters can be chosen that determine the shape of the distribution. The work can be regarded as a first step toward an accepted method for creating inhomogeneous distributions with well-defined stochastic properties.

As a second step in our work about inhomogeneous distributions, we asked "How should we define the level of inhomogeneity in a spatial distribution?". This question is discussed in [5] proposing an objective measure for the level of inhomogeneity in a node distribution. This inhomogeneity measure is of practical use in simulations. It enables us to create random node distributions that reflect the expected real environment in a way that the level of inhomogeneity is the same in both simulation and reality.

At this point, important questions arise for mobile networks: If nodes are placed inhomogeneously, what will happen to the inhomogeneity if they start moving according to some mobility model? Do commonly used random mobility models destroy the inhomogeneity of a distribution? If so, how can we retain the level of inhomogeneity in a mobile wireless network?

The paper at hand finds answers to these questions, thus being a further step toward a richer set of modeling tools for the simulation of sparse wireless networks. In Section II, we revisit the inhomogeneity measure defined in our previous work. Section III investigates the impact of three commonly used mobility models on the inhomogeneity over time. We observe that the random direction model [6], the random waypoint model [7], and the reference point group model [8] all do not retain the inhomogeneity of a starting distribution. Next, in Section IV, we study a modified RWP model that is able to preserve the average inhomogeneity of a starting distribution. Besides being of theoretical interest, we believe that such a "retaining property" also occurs in the real world. For example, students on a university campus gather in lecture halls leading to an inhomogeneous distribution. After a lecture, they move randomly to new locations. In this step the distribution probably becomes less inhomogeneous; once a new lecture starts, however, a similar inhomogeneity value as in the beginning may be observed. Finally, Section V presents related work.

## II. Inhomogeneity Measure

In [5] we define an objective measure for the inhomogeneity of spatial distributions. Some examples for different distributions are given in Figure 1. The proposed measure is based on the continuous refinement of a segmentation of the simulation area $A$ containing $n$ nodes. For each segmentation the actual numbers of nodes $m_i$ in each subarea $A_i$ are compared to the expected number, and the absolute deviations are accumulated. Then, we calculate a weighted sum over all segmentations yielding the final inhomogeneity value

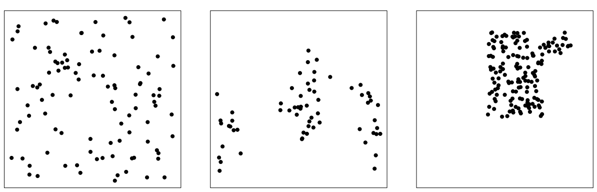
$$h = \frac{1}{2n} \sum_{j=1}^{r} w^{1-j} \max_{(x,y)} \sum_{i=1}^{2^{2j}} \left| m_{i,(x,y)} - \frac{n}{2^{2j}} \right| .$$

In order to achieve the same inhomogeneity value $h$ for distributions which only differ in e.g. movement, mirroring, etc. we consider all possible offsets $(x, y)$. All nodes are moved $x$ length units in horizontal direction and $y$ length units vertically. If a node leaves $A$ it reenters on the opposite edge. We pick the offset that maximizes the local deviation for each subdivision. The number of nodes in the $i^{th}$ subarea for a given offset is called $m_{i,(x,y)}$.

The weight $w = 0.5 \cdot (5 + \sqrt{21})$ is chosen in a way that the measure is normalized on the interval $[0, 1]$ with $0$ signifying a optimal grid distribution and $1$ absolute inhomogeneity with all nodes at the same position.

When at most one node is located in each subarea the calculation is complete. In the formula this upper limit is denoted by $r$.

In [5] we also show that the proposed inhomogeneity measure well fits the human perception of inhomogeneity, which was gained via an online survey.



(a) $h = 0.24$ (b) $h = 0.54$ (c) $h = 0.91$

Fig. 1. Inhomogeneity values for different distributions

## III. Impact of Random Mobility on Inhomogeneity

### A. Mobility Models

There is a rich set of models that can be used to describe the movement of users and devices in wireless networks. On a coarse level, mobility models can be categorized into random models, models with temporal or spatial dependency, and models with geographic restriction.

The two most commonly used models are the random waypoint (RWP) model [7] and the random direction (RD) model [6]. Since there are different variants of these models, we will briefly explain our implementations.

A node moving according to the RWP model can be described by a stochastic process $i \mapsto \{P_i, V_i, T_{p,i}\}$ with the time index $i \in \mathbb{N}$. For each $i$, a node chooses a location ("waypoint") $P_i$ inside the simulation area and moves with a constant speed $V_i$ toward this waypoint. When the waypoint is reached, the node rests for a pause time $T_{p,i}$ and then starts over. The waypoints are chosen from a two-dimensional uniform distribution. In our RWP implementation, a node chooses the speed from a uniform distribution, i.e. $V_i \in [v_{min}, v_{max}]$, and it uses always the same pause time $T_{p,i} = T_p \; \forall i$. Each node moves independently from other nodes.

A node moving according to the RD model is generally described by a stochastic process $i \mapsto \{\Phi_i, V_i, T_i, T_{p,i}\}$. A node chooses a direction angle $\Phi_i$, it then moves with speed $V_i$ for a certain movement time $T_i$, pauses for a period $T_{p,i}$ and then starts over. In our RD implementation, the speed is again chosen from a uniform distribution, the movement time is set to a fixed value (i.e. $T_i = \tau \; \forall i$), and the pause time is always zero. Each node moves independently from other nodes.

Mobility models with spatial dependency are well-suited when users move in groups (e.g. firefighters, military). We use the reference point group mobility (RPG) model [8] here. It can be described as follows: some reference nodes (group leaders) move according to the RWP model. The group members and their waypoints are uniformly distributed on a disk with fixed radius around their group leader. Furthermore, their velocities are set in a way that they arrive at their next waypoint at the same time as their group leader does.

### B. Border Behavior

The behavior of nodes at the border of the simulation area has significant impact on the resulting spatial node distribution [9]. We use two different approaches in our simulation.

The first approach uses impenetrable borders. Using the RD model, a node might hit a border and then gets repelled similarly to the reflection of light on a mirror. Using the RWP model, nodes can never hit the border as they choose the shortest path to their next waypoint, which on average leads them through the center of the simulation area.

The second approach uses penetrable borders. If a node leaves the simulation area on one side it reappears on the opposite side. This can be implemented by simulating on a torus surface [10]. In all models nodes can follow shortest paths across borders.

### C. Simulation Setup

Starting from an initial node distribution — which is supposed to represent a distribution with desired inhomogeneity — a mobility model is applied to all nodes. From time to time, we take a snapshot of the distribution and analyze it using two measures:

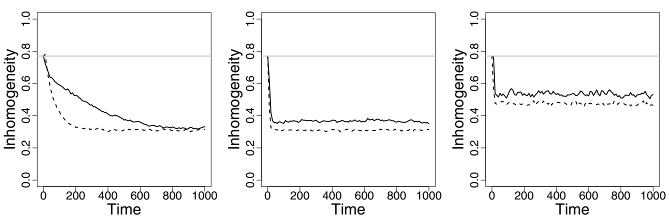
- the inhomogeneity measure $h$ [5] and
- the percentage of border and center nodes.

The percentage of border and center nodes is defined as follows: The simulation area is partitioned into three concentric zones: a border zone occupying 25 % of the simulation area, a center zone occupying 25 % of the simulation area, and a middle zone. A node is considered to be a border node (center node) if it lies inside the border zone (center zone). These values help us in exposing the behavior of mobility models with respect to (unintentional) steady node clusters in the center and at the borders.

The simulation parameters are as follows: 100 nodes are placed on an area of size $10 \times 10$ length units according to a spatial distribution with inhomogeneity $0.77$. All nodes move with a given mobility model. The speeds are chosen from the interval $[0, 0.5]$ length units per time units for RD and $[0.01, 0.5]$ for RWP and RPG. The pause time in the RWP and RPG model is $T_p = 2$ time units. In the case of RPG mobility, 10 group leaders each with 10 group members are used. The simulation time is 1 000 time units. The inhomogeneity and border node measures are determined every 10 time units. All simulations are performed 50 times for a given mobility model, and the resulting measures are averaged.

### D. Simulation Results

*1) Inhomogeneity:* Figure 2 depicts the averaged node inhomogeneity of all three models over time, starting from the inital distribution (gray horizontal line). The solid and dashed curves show the inhomogeneity with impenetrable and penetrable borders, respectively. In all examples, the initial inhomogeneity cannot be maintained by the mobility model but decreases and finally tends toward an asymptotic value.




(a) RD (b) RWP (c) RPG

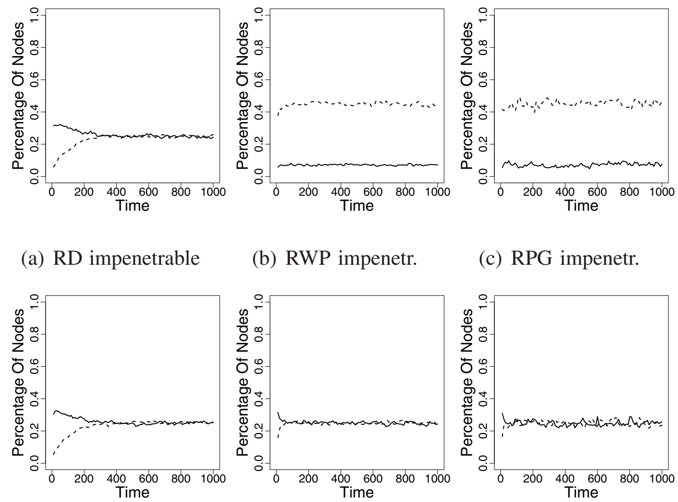
Fig. 2. Inhomogeneity of nodes with random mobility over time

Let us first discuss the results with impenetrable borders (solid curves). Using the RD model, the nodes slowly converge toward a spatial distribution with $h = 0.29$, which is a rather uniform distribution. Using the RWP model, the node distribution converges faster because each node moves toward an independently and uniformly distributed destination. However, RWP results in a spatial distribution with a higher inhomogeneity. This can be explained by the well-known fact that the RWP model produces a higher average node density in the center [9]. The inhomogeneity of nodes moving according to the RPG model is much higher than for the other two models. As with the RWP model, the inhomogeneity value converges very quickly.

Using a penetrable border (dashed curves), the inhomogeneity of nodes moving with RD mobility converges faster; the asymptotic inhomogeneity value is the same as with impenetrable borders. However, using the RWP model on an area with penetrable borders leads to a spatial distribution that has a similar inhomogeneity as the RD model. Also with the RPG model penetrable borders result in faster convergence and a lower inhomogeneity value.

*2) Percentage of Border and Center Nodes:* Figure 3 depicts the percentage of nodes over time in the border zone (solid curve) and in the center zone (dashed curve), respectively.

With RD mobility, the number of nodes at the border and in the center are similar, except at the beginning where the inhomogeneous initial distribution has an impact on it. In the case of RWP mobility, the percentage of nodes in the center zone is much higher than in the border zone. This well-known effect does, however, not occur if we employ penetrable




(a) RD impenetrable (b) RWP impenetr. (c) RPG impenetr.

(d) RD penetrable (e) RWP penetrable (f) RPG penetrable

Fig. 3. Percentage of nodes in the border and center zones over time

borders. The same effect can be observed with RPG. Since with this model whole clusters are moving, the variance in the results is higher.

## IV. Inhomogeneous RWP Mobility Model

In this section we first investigate desired properties of a mobility model which preserves average node inhomogeneity. Then we propose such a mobility model and finally investigate its performance via simulations and compare it to the previously discussed random models.

### A. Desired Properties

A mobility model used in simulations should fulfill some properties to gather simulation results which compare to real world applications. Those properties depend to some degree on the scenario to be simulated. For the simulation of sparse, delay-tolerant networks an inhomogeneous distribution of nodes may be required.

Hence, desirable properties of a random mobility model used in such scenarios are as follows:

- The model supports pause times or idle nodes, to reflect scenarios in which only a fraction of the users are moving at a given time.
- The model leads to an inhomogeneous node distribution, whose average inhomogeneity measure can be tuned (e.g. it maintains the initial degree of inhomogeneity).
- Nodes do not remain in their group (like in RPG) but move to different clusters from time to time. This property has major impact on simulation results as clusters may not be connected with each other per se. Thus, nodes moving between clusters can act as relays or message ferries.
- The model is applicable to areas with both impenetrable and penetrable borders.

### B. Mobility Model

We introduce a mobility model that fulfills these properties. It is basically a modification of the RWP model. The destination waypoints are not generated independently from each other from a uniform distribution but are inhomogeneously distributed. We apply, for example, the method proposed in [4] to generate such a waypoint distribution. Using the inhomogeneity measure from [5] it can be ensured that the inhomogeneity value is approximately at a desired value. Each node moves from its initial position to a waypoint. When all nodes have arrived, the process starts over by generating a new waypoint distribution. The model is called *inhomogeneous random waypoint (IRWP) model*. There are two main issues which need to be resolved.

*1) Destination Waypoint Mapping:* The first question is "How to map nodes from one waypoint distribution to the next?" One simple method is to map nodes to the destination waypoint which is closest to the current waypoint. This results in small movements. These can, e.g., represent intra-group movements or even nodes not actually moving but just jittering. Another simple method is to map nodes randomly to their destination waypoints. Using this method, nodes move further than with the first method.

A combination of these two methods can be used to model a set of nodes that comprise both *idle nodes* (moving only small distances, slowly) and *traveling nodes* (moving larger distances, quickly). To achieve this behavior, a certain percentage of nodes is mapped to their nearest destinations while the remaining nodes are mapped randomly.

*2) Speed and Pausing Behavior:* The second question is "How to choose speed and pause times?". With our model the movement of the nodes is synchronized, i.e. they move from one destination distribution to the next. This restriction is necessary to guarantee that the desired inhomogeneity is approximated.

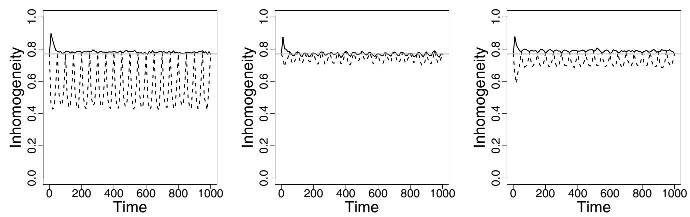
If no real pause times are required, the speeds of the nodes can be calculated depending on the length of their path. Nodes with a close-by destination move very slowly and almost appear jittering, while nodes with a more remote waypoint move faster. If, however, pause times are desired then there are two possibilities to introduce them into our model:

1) Random pause times are injected into the movement of each node. I.e. each node arrives after $t$ time units at the destination but spends any $T_p$ in between waiting. With this approach the nodes' idle times are unsynchronized as with RWP. Only their direction and speed changes are performed in a synchronized fashion. This could also easily be overcome by introducing another random event but would yield a more complicated model.
2) Nodes move with a random speed which is independent of the distance of their destination. When a node reaches its destination it waits until all other nodes have also arrived. In this case the movements are synchronized.

### C. Simulation Setup and Results

The same simulation setup and initial distributions as described in Section III-C are used. In addition, every $50$ time units a new destination waypoint distribution is generated. The three destination waypoint mapping methods (i.e. random waypoint, nearest waypoint, and combined) are compared. In case the combined method is used, $70\,\%$ of the nodes are mapped to the nearest waypoint, the other $30\,\%$ are mapped randomly.

*1) Inhomogeneity:* Figure 4 shows the average inhomogeneity over time in a similar way as in Figure 2. As above, the gray horizontal line indicates the inhomogeneity of the initial distribution.




(a) Random (b) Nearest (c) Combined

Fig. 4. Inhomogeneity of the IRWP model with different destination waypoint mappings

In general, starting from the initial distribution (gray horizontal line), the inhomogeneity value changes since all nodes are moving toward their new destinations. Once all nodes have arrived, the inhomogeneity again approaches its desired (inital) value. The amplitude of the deviations between waypoint distributions depends on the mapping function and the border behavior. In this example, $20$ destination waypoint distributions with similar inhomogeneity were created and used for all three mapping methods.

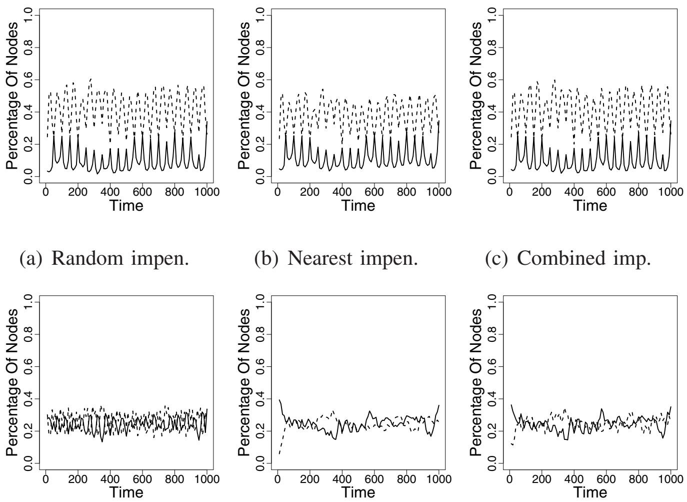
Figure 4(a) shows the inhomogeneity over time using the random mapping method. Using penetrable borders (dashed curve), the distribution becomes very uniform between two waypoints and results in high deviations from the desired inhomogeneity. In fact, the mean inhomogeneity becomes $0.54$. Using impenetrable borders (solid curve), the initial inhomogeneity is much better retained.

If nodes are mapped to their nearest waypoint (see Figure 4(b)), the deviations from the initial inhomogeneity decrease. The use of penetrable borders no longer has a distinct influence since nodes are mapped to close-by destinations and thus do not have to travel across the center. The mean inhomogeneity only changes from $0.77$ to $0.74$ when penetrable borders are used. The main disadvantage here is the low mobility.

The results from the combined method are shown in Figure 4(c). We observe a higher deviation from the initial inhomogeneity with the nearest waypoint method but a lower deviation with the pure random mapping. The mean inhomogeneity in this case is $0.79$ with impenetrable borders and $0.72$ with penetrable borders.

While the RD, RWP, and RPG models were not able to maintain the desired inhomogeneity over time (Figure 2), these results show that the IRWP model maintains the inhomogeneity much better.

*2) Percentage of Border and Center Nodes:* Figure 5 shows the percentage of nodes over time in the border zone (solid curve) and in the center zone (dashed curve), respectively. As is the case with RWP and RPG, also the IRWP model tends to accumulate nodes in the center if impenetrable borders are used (Figs. 5(a)-5(c)). Using penetrable borders, however, this effect can be avoided (Figs. 5(d)-5(f)). Especially with random mapping using penetrable borders the node clusters distribute quite uniformly between center and border over time.




(a) Random impen. (b) Nearest impen. (c) Combined imp.

(d) Random penetr. (e) Nearest penetr. (f) Combined pen.

Fig. 5. Number of nodes in the center and at the borders over time

Summarizing the results, it can be said that:

- IRWP preserves a desired inhomogeneity over time.
- Deviations from the mean inhomogeneity can be controlled via the mapping method and model parameters.
- IRWP behaves similar to RWP and RPG with repect to node accumulations in the center zone.
- If the average inhomogeneity should be approximated, a slightly higher waypoint inhomogeneity has to be chosen, since the movement in between waypoints in general makes the distribution more uniform.

## V. Related Work

The paper [11] presents a modified RWP model. Instead of a uniform waypoint distribution, the author uses a non-uniform probability density. For a given inhomogeneity it is, however, difficult to derive such a density. A similar approach is presented in [12]. In that approach the destination location depends on the current location and time, which leads to the same drawbacks.

The paper [13] proposes a variation of the RD model, called "RD with Location Dependent Parametrization". In this model the new direction of each node depends on its current position. The parametrization of the model can be obtained from existing mobility patterns by applying the conversion model which is also discussed. Due to the fact that this parametrization is determined before the simulation starts and is not changed afterwards, the cluster positions and shapes are fixed. This is not the case with the IRWP model.

A mobility model to control the degree of inhomogeneity is presented in [14]. Here, nodes are distributed on the simulation area in a way that nodes are more likely to be placed into regions that already contain more nodes. During simulation, nodes again tend to move to subareas that are higher populated than to others. The major disadvantage of this approach is that the degree of inhomogeneity cannot be exactly predefined.

## VI. Conclusions

In this paper we investigated the impact of three well-known mobility models on the node inhomogeneity over time. Furthermore, for the same models we analyzed the percentage of nodes in the border and center zones of the simulation area. Motivated by the need for a mobility model that preserves inhomogeneities, we proposed and analyzed a modified random waypoint model (IRWP) that maintains the inhomogeneity of an initial node distribution.

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## References

[1] D. Avidor and S. Mukherjee, "Hidden issues in the simulation of fixed wireless systems," *ACM Wireless Netw.*, vol. 7, pp. 187–200, Apr. 2001.
[2] R. Vilzmann, J. Widmer, I. Aad, and C. Hartmann, "Low-complexity beamforming techniques for wireless multihop networks," in *Proc. IEEE SECON*, (Reston, VA, USA), 2006.
[3] X. Liu and M. Haenggi, "Toward quasiregular sensor networks: Topology control algorithms for improved energy efficiency," *IEEE Trans. on Parallel and Distributed Systems*, vol. 17, pp. 975–986, 2006.
[4] C. Bettstetter, M. Gyarmati, and U. Schilcher, "An inhomogeneous spatial node distribution and its stochastic properties," in *Proc. ACM/IEEE MSWiM*, (Chania, Crete Island, Greece), 2007.
[5] U. Schilcher, M. Gyarmati, C. Bettstetter, Y. W. Chung, and Y. H. Kim, "Measuring inhomogeneity in spatial distributions," in *Proc. IEEE Veh. Techn. Conf.*, (Marina Bay, Singapore), 2008.
[6] C. Bettstetter, "Mobility modeling in wireless networks: Categorization, smooth movement, and border effects," *ACM Mobile Computing and Commun. Rev.*, vol. 5, pp. 55–67, July 2001.
[7] J. Broch, D. A. Maltz, D. B. Johnson, Y.-C. Hu, and J. Jetcheva, "A performance comparison of multi-hop wireless ad hoc network routing protocols," in *Mobile Computing and Networking*, pp. 85–97, 1998.
[8] X. Hong, M. Gerla, G. Pei, and C.-C. Chiang, "A group mobility model for ad hoc wireless networks," in *Proc. ACM MSWiM*, (New York, NY, USA), pp. 53–60, 1999.
[9] C. Bettstetter and O. Krause, "On border effects in modeling and simulation of wireless ad hoc networks," in *Proc. IEEE Intern. Conf. on Mobile and Wireless Commun. Networks (MWCN)*, (Recife, Brazil), Aug. 2001.
[10] N. A. C. Cressie, *Statistics for Spatial Data*, ch. 8.2.5. Wiley, 1991.
[11] P. Lassila, "Spatial node distribution of the random waypoint mobility model with applications," *IEEE Trans. Mobile Comput.*, vol. 5, no. 6, pp. 680–694, 2006.
[12] W. Hsu, K. Merchant, H. Shu, C. Hsu, and A. Helmy, "Weighted waypoint mobility model and its impact on ad hoc networks," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 9, no. 1, pp. 59–63, 2005.
[13] B. Gloss, M. Scharf, and D. Neubauer, "A more realistic random direction mobility model," in *Proc. COST 290 Management Committee Meeting*, (Würzburg, Germany), Oct. 2005.
[14] S. Lim, C. Yu, and C. R. Das, "Clustered mobility model for scale-free wireless networks," in *Proc. IEEE Local Computer Networks*, (Tampa, FL, USA), pp. 231–238, Nov. 2006.