Non-Colliding First Messages in Slotted ALOHA: Further Insights Toward a Practical Solution

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***Abstract*—Consider n nodes competing for access on a channel using slotted ALOHA. Our aim is to maximize the probability Φ that the first message within s slots does not collide. We derive an expression for the transmit probabilities in each slot, maximizing Φ. As opposed to previous work, the expression is non-recursive, thus easier to calculate and more convenient for practical implementations. Furthermore, we address the problem that, in practical applications, the number of competing nodes n is likely to be unknown and has to be estimated by each node. We study the sensitivity of channel access scheme with respect to deviations from the actual n via simulations. It is shown that overestimation of n is a better strategy than underestimation.**

***Index Terms*—Medium access control, ALOHA, first message, collision probability, random access**

I. Introduction and Motivation

Node selection is one of the most basic problems in distributed systems: one out of many nodes should be selected in a distributed manner to undertake a certain task. Various algorithms and theorems have been proposed for different systems and modeling assumptions, designed to optimize different performance criteria (see, e.g., [1]). Also in wireless communication systems, node selection is an important building block in many scenarios. It is needed, for instance, for data processing techniques in sensor networks to choose a "data gathering node" [2] and for cooperative diversity techniques in relaying networks to choose a "relay node" [3].

A straightforward approach for node selection in wireless systems is as follows: A node broadcasts a query message to all adjacent nodes, indicating its wish that a node should be selected, and indicating the criterion that qualifies a node to serve as a selected node. All nodes receiving this query message and fulfilling the criterion now compete for random access on the shared medium. The node that successfully accesses the medium *first* — transmitting a reply message to the querying node — wins the selection process and acts as selected node in the following.

As such a *first message* in the reply phase of a selection process plays a special role, it is desired that this message does not collide with other (first) messages [4]. In a previous paper [5], we performed a theoretical analysis on the occurrence of a non-colliding first message. Given a slotted shared medium, each of n nodes may transmit in a slot using ALOHA without carrier sensing. We derived a medium access strategy that maximizes the probability that, within s slots, there occurs a first message which does not collide. The optimal solution represents a slow start strategy: each node transmits with low probability in the first slot, it then gradually increases its access probability, until it would access the last slot with probability $1/n$.

Putting this optimal slot access strategy into practice raises some important questions:

- First, the computation of the optimum transmit probabilities requires each node to know the number n of nodes participating in the channel access competition on the shared medium. If this number is unknown, a node has to estimate it. An open research issue is thus to determine the impact on the performance due to errors in this estimation. The paper at hand investigates this issue, analyzing the sensitivity of the non-colliding first message probability as a function of the relative estimation error. It is shown that significant estimation errors lead to only small performance degradations, and that in general an overestimation of n is better than an underestimation.
- Second, the calculation of the optimal slot access strategy is based on a recursive equation that depends on n in each recursion. This means the following: if the number of nodes changes, all s recursions must be recalculated. The paper at hand derives an alternative expression for the optimal transmit probabilities that enables us to store them in a table, where the values are independent from the number of nodes n.

We organize the remainder of this paper as follows: Section II states our modeling assumptions and definitions. Section III derives an estimation Φ^* of the original expression for calculating the probability Φ of a non-colliding first message. Section IV answers the question which transmission probabilities have to be chosen by each node to maximize Φ. Section V investigates the sensitivity of Φ to estimation errors with respect to the number of competing nodes. Finally, Section VI addresses related work.

II. Modeling Assumptions and Definitions

Consider n nodes competing for random access on a channel following a slotted ALOHA scheme without carrier sensing [6]. Further assume, that there are s slots, each of these slots has the same duration τ and a message can be transmitted in the course of one slot.

Let p_{ij} denote the probability that node j transmits a message in slot i, with $j \in \{1, ..., n\}$ and $i \in \{1, ..., s\}$. The sending probabilities can be expressed with a matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \dots & \dots & \dots & \dots \\ p_{s1} & p_{s2} & \dots & p_{sn} \end{bmatrix} = [\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n]$$

where the jth column represents the vector of sending probabilities of node j, denoted as $\vec{p}_j = (p_{1j}, p_{2j}, \dots, p_{sj})^{\mathrm{T}}$. If we assume that all nodes behave statistically the same we can skip the index j and write $p_i := p_{ij} \, \forall j$. Therefore, all columns of the corresponding matrix $\mathbf{P}$ are equal. Hence, for this case, we denote the sending probabilities with $\vec{p} = (p_1, p_2, \dots, p_s)^{\mathrm{T}}$.

If two or more nodes transmit during the same slot, a message collision occurs at the receiver. In this case, we assume that none of the messages can be decoded correctly. A message that does not collide is called a *non-colliding* message.

A slot is empty if no node transmits during this slot. The first non-empty slot is the slot i in which at least one message is sent while the previous slots $1, ..., i-1$ are empty. A message sent in the first non-empty slot is called a *first message*.

III. Probability of a Non-Colliding First Message

The number of messages sent in a given slot i can be described by a random variable M_i. A message sent in slot i does not collide if exactly one node transmits during this slot; the probability for this event is denoted by $\mathbb{P}[M_i = 1]$. A message sent in slot i is the first message to be sent if there was no message in previous slots; the probability for this event is $\prod_{w=1}^{i-1} \mathbb{P}[M_w = 0]$ for $i > 1$. Thus, if we assume that all nodes behave statistically the same, the probability that within s slots occurs a first message that does not collide is [5]

$$\begin{aligned} \Phi(n, s, \vec{p}) &= \\ &= \mathbb{P}[M_1 = 1] + \sum_{i=2}^{s} \left(\prod_{w=1}^{i-1} \mathbb{P}[M_w = 0] \right) \cdot \mathbb{P}[M_i = 1] \, . \end{aligned} \quad (1)$$

With our assumptions, the probability that k messages are sent in a given slot i is given by the binomial distribution

$$\mathbb{P}[M_i = k] = \binom{n}{k} \cdot p_i^k \cdot (1 - p_i)^{n-k}$$

with $k \in \{0, \dots, n\}$. It is possible to approximate this binomial distribution by a Poisson distribution

$$\mathbb{P}[M_i = k] \approx \frac{\lambda_i^k}{k!} \cdot \exp(-\lambda_i) \quad (2)$$

with parameter $\lambda_i = n \cdot p_i$ for large enough n and small p_i. Due to the fact that $\Phi(n, s, \vec{p})$ is composed of independent random variables, we can apply (2) in (1) and obtain

$$\Phi^*(n, s, \vec{p}) = \sum_{i=1}^{s} \exp\left(- \sum_{w=1}^{i-1} \lambda_w \right) \cdot \lambda_i \cdot \exp(-\lambda_i).$$

It follows, that for sufficiently many nodes n and small p_i, we can write

$$\Phi(n, s, \vec{p}) \approx \Phi^*(n, s, \vec{p}) \, . \quad (3)$$

IV. Maximizing the Probability of a Non-Colliding First Message

A. Derivation of sending probabilities

To maximize the probability of a non-colliding first message, the sending probabilities have to be chosen appropriately. An expression for calculating these optimal sending probabilities is [5]

$$p_{s-k} = \frac{1}{n} \cdot \frac{(n-1)^k - n \cdot \alpha_k \cdot \beta_k}{(n-1)^k - \alpha_k \cdot \beta_k} \quad (4)$$

for all $k \in \{0, \dots, s-1\}$ with $\alpha_k := \left(\frac{(n-1)^k}{n} \right)^n$ and the recursively defined term

$$\beta_k := \begin{cases} 0 & \text{for } k = 0 \\ \left((n-1)^{k-1} - \alpha_{k-1} \cdot \beta_{k-1} \right)^{1-n} & \text{else.} \end{cases}$$

The key observation for deriving this expression is that the optimal transmission probability for slot $(s-k)$ on a channel with s slots is the same as the optimal transmission probability of the first slot on a channel with $(k+1)$ slots [5].

Applying this observation to Φ^*, the optimal sending probabilities for maximizing Φ^* are calculated as

$$p_{s-k}^* = \frac{1}{n} \cdot \gamma_k, \quad (5)$$

with the recursively defined term

$$\gamma_k := \begin{cases} 1 & \text{for } k = 0 \\ 1 - \exp(-\gamma_{k-1}) & \text{else.} \end{cases}$$

The first nine values of γ_k are shown in Table I.

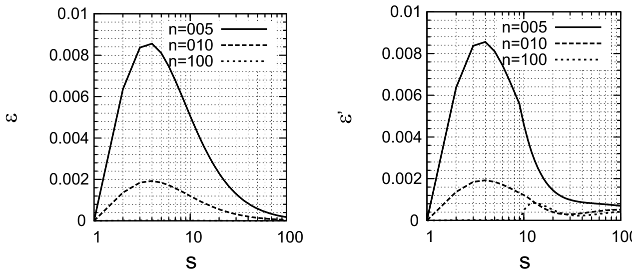
TABLE I
Values of γ_k.

k	0	1	2	3	4	5	6	7	8
γ_k	1	.632	.468	.374	.312	.268	.235	.209	.189

Due to Equation (3), the optimal sending probabilities $\vec{p}^* = (p_1^*, \dots, p_s^*)$ of Φ^* serve as an approximation for the optimal sending probabilities $\vec{p} = (p_1, \dots, p_s)$ of Φ. Fig. 1(a) illustrates the relative error $\varepsilon = |\Phi(n, s, \vec{p}) - \Phi(n, s, \vec{p}^*)| / \Phi(n, s, \vec{p}) \cdot 100\%$ if probabilities $\vec{p}^*$ are used instead of probabilities $\vec{p}$. For $n \geq 5$ the error is below 0.01%.

B. The term γ_k is independent of n

The recursive term β_k of (4) depends on n while the recursive term γ_k of (5) is independent of n. For practical applications the independency of γ_k from n is an important advantage as it removes the need to recalculate the recursive function whenever n changes. The independency of γ_k from n allows us to precalculate and store values of γ_k up to a certain value s_{max}. In that way, the sending probabilities can be easily calculated for all systems with at most $s_{max}+1$ slots and an arbitrary number of nodes.



(a) The relative error in percent of Φ with optimal sending probabilities $\vec{p}$ from Φ with sending probabilities $\vec{p}^*$.

(b) The relative error in percent of Φ with optimal sending probabilities $\vec{p}$ from Φ with approximated sending probabilities $\hat{\vec{p}}$.

Fig. 1. Approximation errors of $\vec{p}^*$ (Fig. 1(a)) and $\hat{\vec{p}}$ (Fig. 1(b)).

C. Approximation of γ_k

The term γ_k is monotonically decreasing, bounded, and converges to zero for $k \to \infty$. Therefore, finding an approximation for it is easier than for β_k. The proofs for these attributes are given below.

Property 1: $\forall k \geq 0: \ \gamma_k \geq 0$.

Proof: For $k = 0$ the statement is obvious. Assume that $\gamma_k \geq 0$ for $k \geq 1$. It follows $\gamma_{k+1} = 1 - \exp(-\gamma_k) \geq 0$. ∎

Property 2: $\forall k \geq 0: \ \gamma_{k+1} \leq \gamma_k$.

Proof: Clearly, $\gamma_1 \leq \gamma_0$ holds. We assume $\gamma_k \leq \gamma_{k-1}$ for $k \geq 1$. Due to the fact that $\gamma_{k+1} = 1 - \exp(-\gamma_k) \leq 1 - \exp(-\gamma_{k-1}) = \gamma_k$ the inequality also holds for $k+1$. ∎

Property 3: $\lim\limits_{k\to\infty} \gamma_k = 0$.

Proof: Since γ_k is monotonically decreasing and bounded, it follows that it converges. Hence, $\lim\limits_{k\to\infty} \gamma_k = \lim\limits_{k\to\infty} 1 - \exp(-\gamma_k) = 1 - \exp(-\lim\limits_{k\to\infty} \gamma_k)$ it follows that $\lim\limits_{k\to\infty} \gamma_k = 0$. ∎

If memory saving is important, then γ_k can be approximated for $k \geq 9$ with

$$\hat{\gamma}_k \approx \frac{1}{0.518 \cdot (k+2)}.$$

Then only the values of γ_k for $k \leq 8$, which are listed in Table I, have to be stored in memory.

Hence, as an approximation for the optimal sending probabilities of Φ, $\hat{\vec{p}} = (\hat{p}_1, \ldots, \hat{p}_s)$ is proposed with

$$\hat{p}_{s-k} = \begin{cases} \frac{1}{n} \cdot \gamma_k & \text{for } k \leq 8 \\ \frac{1}{n} \cdot \hat{\gamma}_k & \text{else.} \end{cases}$$

Fig. 1(b) shows the relative approximation error in percent $\varepsilon' = |\Phi(n,s,\vec{p}) - \Phi(n,s,\hat{\vec{p}})|/\Phi(n,s,\vec{p}) \cdot 100\%$. Again, the error is well below 0.01% for $n \geq 5$.

V. Sensitivity of Φ with respect to the number of estimated nodes

The number of nodes n competing for random access on the shared channel must be known to calculate the optimal sending probabilities. In practical applications, however, it is unlikely that n is exactly known and thus has to be estimated. In this section, the impact of estimation errors (i.e., overestimation or underestimation) on the probability of a non-colliding first message is discussed. In the following, let n denote the *actual* number of nodes that compete for random access on the shared medium. Furthermore, let $\vec{\delta} = (\delta_1, \ldots, \delta_n)^{\mathrm{T}}$ represent the vector of estimation errors of the nodes. Each δ_j can be either positive (overestimated n), negative (underestimated n), or zero if there is no estimation error.

As each of the nodes has to estimate the potential number of competing nodes on its own, sending probabilities of nodes can differ from each other. This means that nodes are not behaving statistically the same anymore. Hence, we use the matrix definition introduced in Section II, i.e., the jth column of $\mathbf{P}$ represents the sending probabilities of node j. To clarify that each node j has calculated its sending probabilities based on its own estimation on n, which possibly introduced an estimation error δ_j, we write $\mathbf{P}_{\vec{\delta}}$. Further, let $\vec{p}_\delta$ denote the optimal sending probabilities for $n+\delta$ nodes and $\vec{p}$ the optimal sending probabilities for n nodes ($\delta = 0$).

The probability of a non-colliding first message, if nodes may behave statistically different, is given by

$$\Phi'(n,s,\mathbf{P}_{\vec{\delta}}) = \\ = \sum_{i=1}^{s} \left(\prod_{w=1}^{i-1} \prod_{t=1}^{n} (1 - p_{wt}) \right) \cdot \left(\sum_{j=1}^{n} p_{ij} \cdot \prod_{\substack{t=1 \\ t \neq j}}^{n} (1 - p_{it}) \right).$$

The term p_{ij} is the sending probability of node j for slot i, calculated for $n + \delta_j$ competing nodes.

If $\delta_j = \delta$ for all j, which means that all nodes are estimating the same number of competing nodes, it is clear that

$$\Phi'(n,s,\mathbf{P}_{\vec{\delta}}) = \Phi(n,s,\vec{p}_\delta). \tag{6}$$

Otherwise, if not all δ_j are equal, two scenarios can occur: First, the probability of a non-colliding first message decreases,

$$\exists \delta_i : \ \Phi'(n,s,\mathbf{P}_{\vec{\delta}}) < \Phi(n,s,\vec{p}),$$

or, second, the probability of a non-colliding first message increases,

$$\exists \delta_i : \ \Phi'(n,s,\mathbf{P}_{\vec{\delta}}) > \Phi(n,s,\vec{p}). \tag{7}$$

The consequence of (7) is, at first glance, surprising as it shows that there are scenarios where estimation errors on the number of competing nodes can in some cases actually lead to an increased probability of a non-colliding first message. This is illustrated in the following example.

Example: On a channel with $n = 5$ competing nodes and $s = 10$ slots the probability of a non-colliding first message is $\Phi(5,10,\vec{p}) = 0.87$ if there are no estimation errors. If the estimation errors are given by $\vec{\delta} = (4,2,3,0,-3)$ then $\Phi'(5,10,\mathbf{P}_{\vec{\delta}}) = 0.88$. In this simple scenario the improvement is due to the fact that one of the nodes (node 5) estimates that there are only two competing nodes. Hence, this node applies high sending probabilities and is thus likely to transmit. All other nodes overestimate the number of competing nodes and therefore use low sending probabilities.

In the extreme case, $\Phi^{'}$ converges to 1 if there is one node which maximally underestimates (i.e. it assumes to be the only node), and all other nodes highly overestimate (i.e., they assume there are many competing nodes). In this case, the node sends with probability 1, while the sending probabilities for all other nodes are very small. Therefore, the probability that these nodes cause a collision with the other node becomes negligible.

A. All nodes either overestimate or underestimate

First, we analyze the impact of estimation errors on the probability of a non-colliding first message, if all nodes either overestimate or underestimate. To begin with, we present two properties that help us in the analysis. Both establish a lower bound for $\Phi^{'}$:

Property 4: If all nodes overestimate, the probability of a non-colliding first message is greater or at least equal to the probability in case all nodes overestimate with maximum estimation error:

$$[\forall j : \delta_j \geq 0] \Rightarrow \Phi^{'}(n, s, \mathbf{P}_{\vec{\delta}}) \geq \Phi(n, s, \vec{p}_{\delta_{max}}),$$

with $\delta_{max} = \max_j \delta_j$.

Proof: From (6) it follows that $\Phi(n, s, \vec{p}) = \Phi^{'}(n, s, \mathbf{P}_{(0,\ldots,0)})$. Further, it follows that $\Phi(n, s, \vec{p}_{\delta_{max}}) = \Phi^{'}(n, s, \mathbf{P}_{(\delta_{max},\ldots,\delta_{max})})$. Since $\Phi(n, s, \vec{p})$ is the maximum of Φ, $\Phi(n, s, \vec{p}) > \Phi(n, s, \vec{p}_{\delta_{max}})$ for $\delta_{max} > 0$. Now, if $\delta_j = \delta_{max} \forall j$, all nodes are sending with too low probabilities. If for some j the error δ_j is decreased, node j will send with higher probabilities, which always results in an increased $\Phi^{'}$. ■

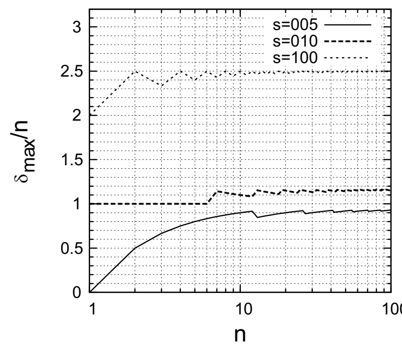
Property 5: If all nodes underestimate by some estimation errors it follows that the probability of a non-colliding first message is greater or at least equal to the probability if all nodes underestimate with maximum estimation error:

$$[\forall j : \delta_j \leq 0] \Rightarrow \Phi^{'}(n, s, \mathbf{P}_{\vec{\delta}}) \geq \Phi(n, s, \vec{p}_{\delta_{min}}),$$

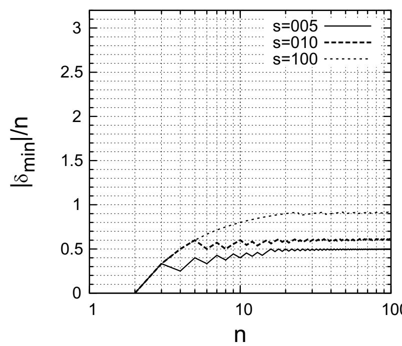
with $\delta_{min} = \min_j \delta_j$.

Proof: The proof is analogous to that of Property 4. ■

As we have found lower bounds for both over- and underestimation, we can plot the worst-case scenario, where all nodes are either over- or underestimating with an estimation error of δ_{max} or δ_{min}. In Fig. 2(a) the ratio δ_{max}/n is plotted for $n = 1, \ldots, 100$ nodes (it is assumed that all nodes are overestimating), where δ_{max} is the maximum value such that the value of $\Phi(n, s, \vec{p}_{\delta_{max}})$ is at most 10% lower than $\Phi(n, s, \vec{p})$. This means that even in the worst case (all nodes overestimate by δ_{max}), the reduction from the optimal value $\Phi(n, s, \vec{p})$ is at most 10%. If there are nodes that have minor estimation errors, the decrease from $\Phi(n, s, \vec{p})$ will be smaller according to Property 4. Fig. 2(b) shows the maximum (under-) estimation error. Again, all nodes underestimate and δ_{min} is the minimum value such that the value of $\Phi(n, s, \vec{p}_{\delta_{min}})$ is at most 10% lower than $\Phi(n, s, \vec{p})$. The worst case occurs if all nodes underestimate by $|\delta_{min}|$ for which the reduction of $\Phi(n, s, \vec{p})$ is still smaller than or equal to 10%. If some nodes commit minor estimation errors this reduction will be smaller (see Property 5). As can be seen in these two figures, in the special case where all nodes either over- or underestimate, δ_{max} is generally considerably larger than $|\delta_{min}|$. If there are, for instance, 10 nodes and 10 slots, then $\delta_{max} = 12$ but $|\delta_{min}| = 6$. Hence, in this example, either all nodes are allowed to overestimate by at most 12 nodes or all are allowed to underestimate by at most 6 nodes if a decrease of the probability of a non-colliding first message of at most 10% is tolerable.



(a) Ratio of maximum overestimation error



(b) Ratio of maximum underestimation error

Fig. 2. Ratio of maximum overestimation error (Fig. 2(a)) and maximum underestimation error (Fig. 2(b)) such that reduction from optimal Φ remains below or equal to 10%.

B. Nodes overestimate or underestimate

The general case where nodes are allowed to over- and/or underestimate is more challenging. Our hypothesis here is, that

$$[\forall j : \delta_{min} \leq \delta_j \leq \delta_{max}] \Rightarrow$$
$$\Rightarrow \Phi^{'}(n, s, \mathbf{P}_{\vec{\delta}}) \geq \min(\Phi(n, s, \vec{p}_{\delta_{min}}), \Phi(n, s, \vec{p}_{\delta_{max}})).$$

In order to verify this hypothesis, we simulate the sensitivity of $\Phi^{'}$. We assume that the estimations of the nodes, denoted as $\hat{n}_j := n + \delta_j$, follow a random variable $\hat{N} := X + 2$, where X is a random variable following a negative binomial distribution with parameters r and p [7]. The purpose of adding 2 to X is that we assume that there are at least two competing nodes. The incentives for choosing this discrete probability distribution as a basis for our simulation are: (a) it is a distribution with two parameters; allowing us to define expectation value and variance, and (b) as δ_i are discrete also the distribution used for simulation should be discrete. Clearly, the expectation value is $\mathbb{E}[\hat{N}] = r \cdot \frac{1-p}{p} + 2$, and the variance is $\mathrm{Var}[\hat{N}] = r \cdot \frac{1-p}{p^2}$.

The probability of a non-colliding first message $\Phi^{'}$ for 100 node selections with random estimations $\hat{n_i}$ following $\hat{N}$, with $\mathrm{Var}[\hat{N}] = (c \cdot n)^2$, is depicted in Fig. 3. We simulate using $n = 10$ and $n = 50$ nodes, $s = 10$ slots, and $c \in \{0.4, 0.7\}$. For $\mathbb{E}[\hat{N}] = n$ and $c = 0.7$, $\Phi^{'}$ decreases and fluctuates. For

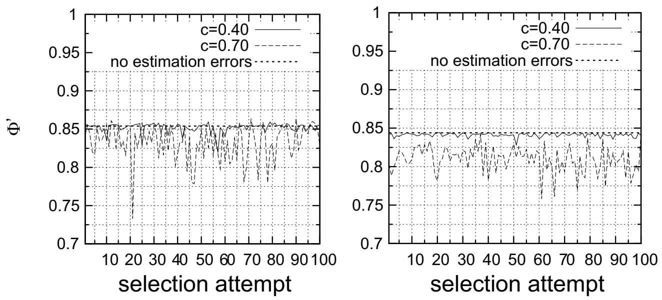


Fig. 3. Simulation for $n = 10$ (left) and $n = 50$ nodes (right) and $s = 10$ slots, $\mathbb{E}[\hat{N}] = n$, $\mathrm{Var}[\hat{N}] = (c \cdot n)^2$.

$c = 0.4$ the values of $\Phi^{'}$ are almost equal to the value without estimation errors. Figure 4 shows the results of the simulation

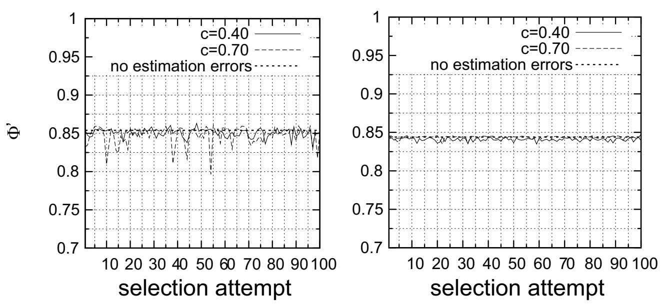


Fig. 4. Simulation for $n = 10$ (left) and $n = 50$ nodes (right) and $s = 10$ slots, $\mathbb{E}[\hat{N}] = 1.3 \cdot n$, $\mathrm{Var}[\hat{N}] = (c \cdot n)^2$.

carried out with $\mathbb{E}[\hat{N}] = 1.3 \cdot n$, which means that there is a tendency to overestimate. As can be seen, the fluctuations for $c = 0.7$ decrease. Therefore, for uncertain estimations the applied estimation algorithm should have the tendency to overestimate rather than to underestimate.

VI. Related Work

In [8] the authors also investigate and optimize the probability that the first channel access does not collide. This paper requires, however, that the sum of all sending probabilities of each node adds up to one. As we do not impose this constraint in our modeling, we achieve a higher probability of a non-colliding first message. The aim in [4] is to select the node that actually has the best value regarding a certain selection metric. The authors propose a function that maps from this metric to a, generally node-dependent, back-off time. With our approach, all nodes meeting a certain threshold have the same chance of selection. The paper [9] derives transmit probabilities for a slotted channel, where the modeling assumption is that after a certain number of slots there must be one selected node. Different to our approach, it does not matter if collisions occur in earlier slots. The papers [10]–[12] investigate and propose approaches to control the number of responses from a set of nodes. First, a node sends out a "feedback request sample" to all other nodes. Each of these nodes chooses a backoff time randomly. If a feedback response of some other node is confirmed by a node during its backoff time, this node does not send its feedback message. If no such response is listened by the node, it sends a feedback response. The difference to our paper is that nodes must overhear possible feedback responses of other nodes.

VII. Conclusions

This paper analyzes two important aspects for the practical usability of the channel access model proposed in [5]. First, an alternative method for calculating the optimal sending probabilities is derived, making an implementation easier. Second, the impact of over- or underestimation of the number of competing nodes is investigated. Simulations show that it is preferable to overestimate rather than underestimate the number of competing nodes.

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