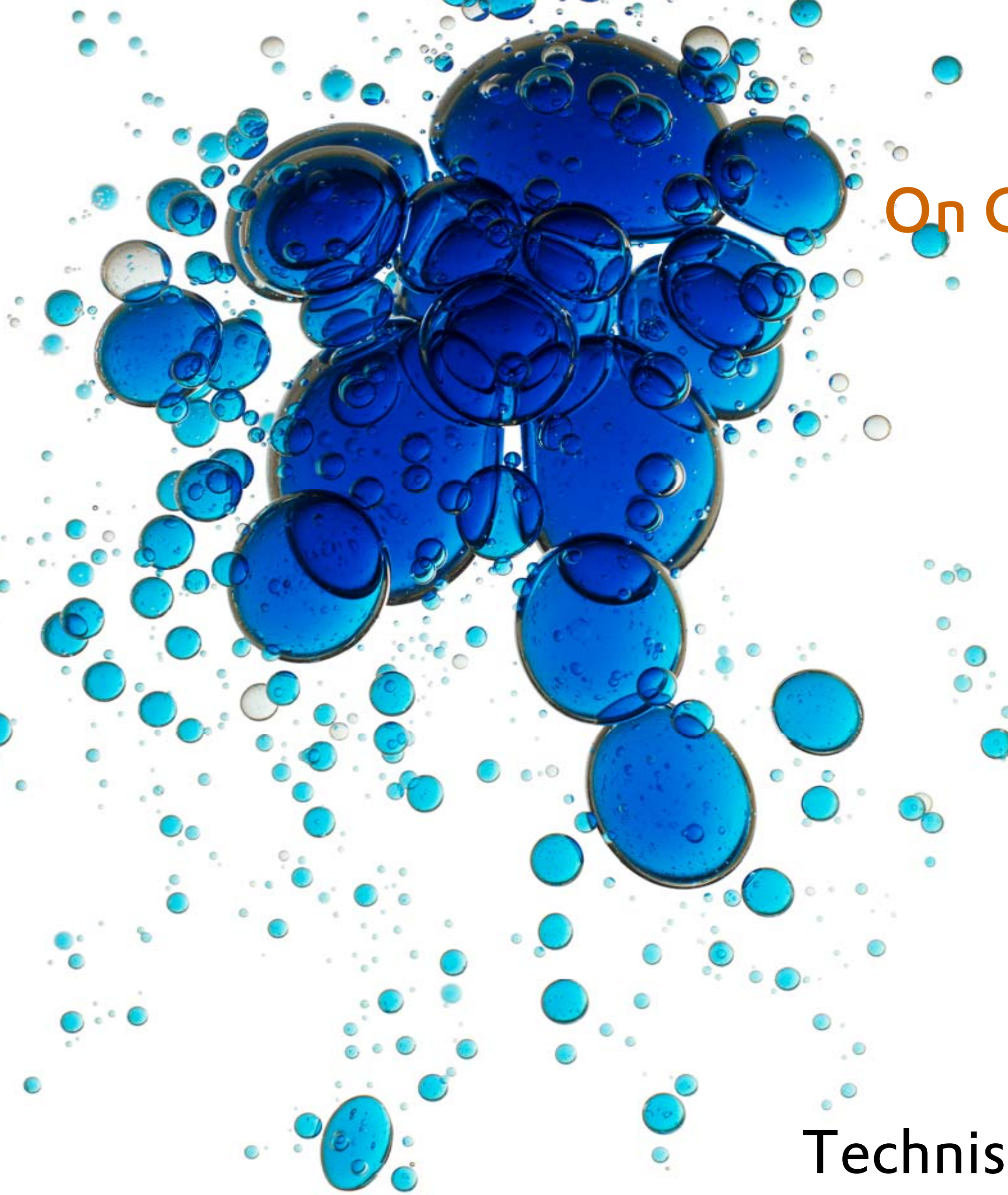


On Colliding First Messages in Slotted ALOHA

Christian Bettstetter

Günther Brandner

Robert Vilzmann

Cannes, Sept 16, 2008

University of Klagenfurt

Lakeside Labs GmbH

Technische Universität München

Motivation

Several techniques in wireless and wired networking require some method for **distributed node selection.**

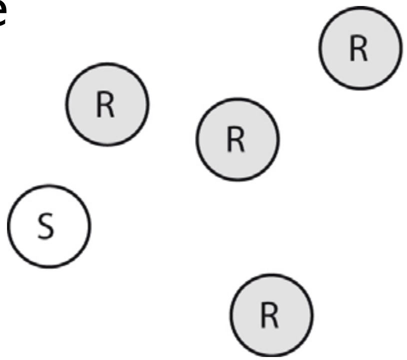
Examples:

- Cooperative relaying techniques in wireless networks to choose a "relay node"
- Data processing techniques in sensor networks to choose a "data gathering node"

A possible way to perform node selection

Step 1: Determine a set of candidate nodes

- A node broadcasts a query message to all neighboring nodes.
- This message indicates a certain criterion (or several criteria) that qualifies to serve as a selected node
- Each receiving node that fulfills the criterion becomes part of a candidate set.

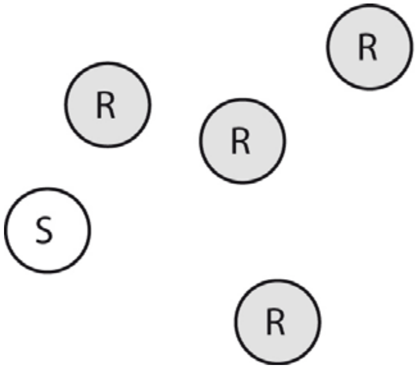


Step 2: Determine the selected node

- All nodes of the candidate set compete for random access on the shared medium, to send back a reply.
- The node that successfully accesses the medium *first* wins the selection process and acts as selected node.

The importance of "first messages"

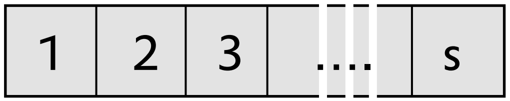
- The reply message of the node answering *first* is more *important* than subsequent reply messages.
- This reply message should *not collide* with other messages, hence only one node should access the channel.



This discussion leads to the following **MAC design issues**:

- What is the probability that there is a first message that does not collide?
- How can we maximize this probability?
- What is the tradeoff between this probability and the delay of the selection process?

Modeling assumptions and definitions



- *n* devices
- *s* slots with slotted ALOHA

Design parameter: Each device transmits with probability p_i in slot i

- If two or more nodes transmit in the same slot, a *message collision* occurs. A message not suffering from a collision is called a *non-colliding message*.
- A slot is *empty* if no node transmits during this slot. The *first non-empty slot* is the slot i in which at least one message is sent while previous slots 1, . . . , i–1 were empty. A message sent in the first non-empty slot is called a *first message*.

Non-colliding first message probability

What is the probability that there occurs a non-colliding first message within *s* slots?

$$\Phi(n, s, p_1, \ldots, p_s) =$$

$$= \quad n\, p_1\, (1 - p_1)^{n-1} \quad + \quad (1 - p_1)^n \cdot n\, p_2\, (1 - p_2)^{n-1} \quad + \quad \ldots$$

Exactly 1 message in slot $i = 1$. *or* No message in slot $i = 1$. *and* Exactly 1 message in slot $i = 2$.

$$= \quad n \sum_{i=1}^{s} \left(\prod_{w=0}^{i-1} (1 - p_w)^n \right) \cdot p_i\, (1 - p_i)^{n-1} \qquad \text{with} \quad p_0 := 0.$$

Lakeside Labs

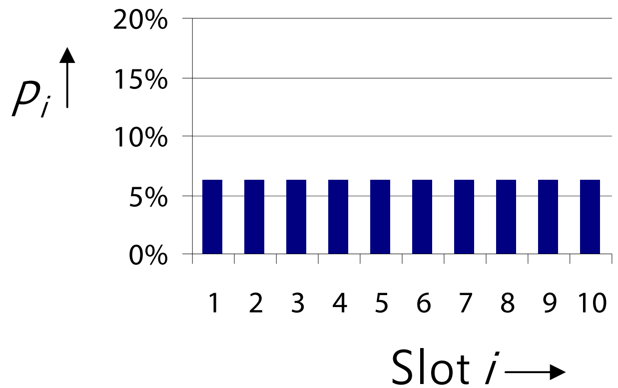
Optimizing the non-colliding first message probability

How to set the transmission probabilities p_i to maximize the probability Φ of obtaining a non-colliding first message within s slots?

Example: $n = 5$ nodes on a channel with $s = 10$ slots

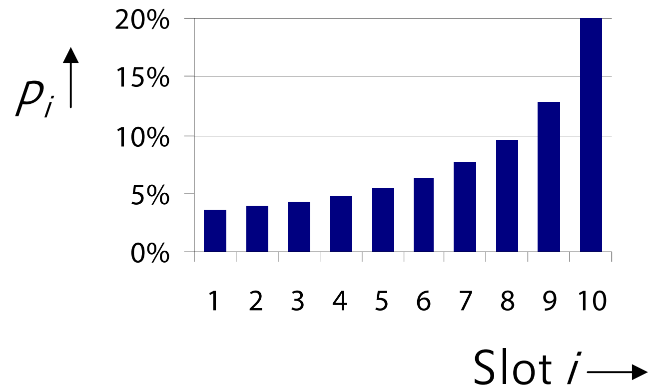
(a) Each slot i has same p_i:

$\Phi_{max} = 84.05\,\%$ obtained with



(b) Each slot i may have different p_i:

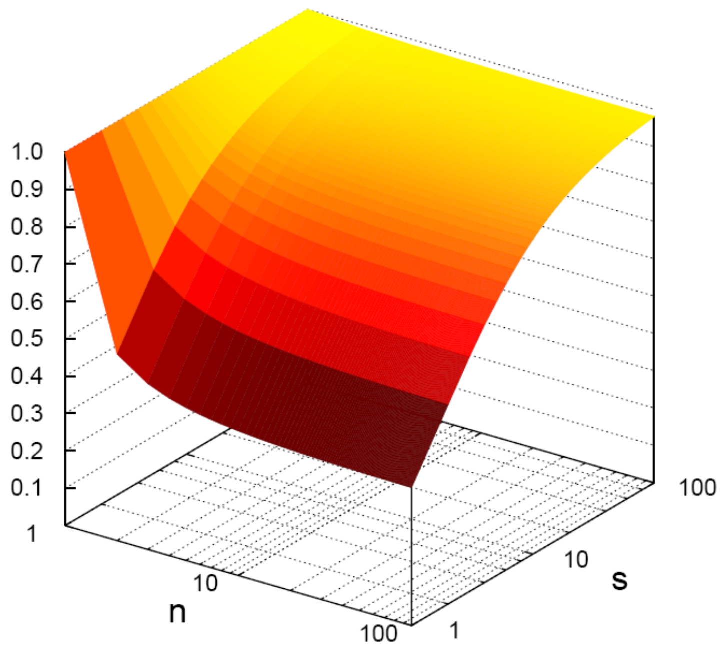
$\Phi_{max} = 86.68\,\%$ obtained with



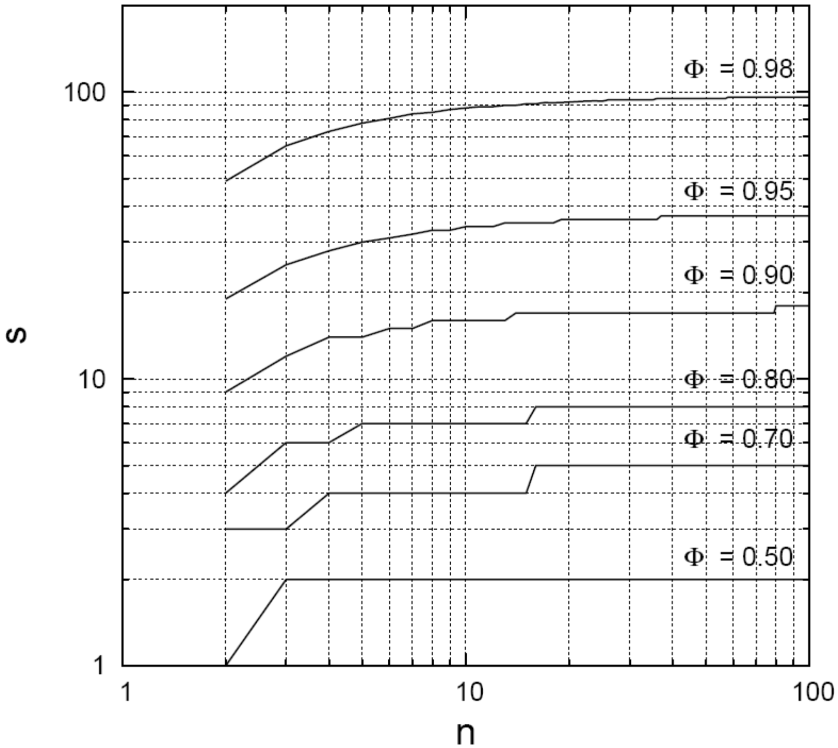
Slow start strategy

Lakeside Labs

Maximum possible non-colliding first message probability



(a) $\max \Phi(n, s, p_1, \ldots, p_s)$



(b) Contour lines of $\max \Phi(n, s, p_1, \ldots, p_s)$

Optimizing the non-colliding first message probability

Probability Φ gets maximum if we set the transmission probabilities to

$$p_{s-k} = \frac{1}{n} \cdot \frac{(n-1)^k - n\,\alpha_k \beta_k}{(n-1)^k - \alpha_k \beta_k}$$ with index $k \in \{0, \ldots, s-1\}$

$$\alpha_k := \left(\frac{(n-1)^k}{n}\right)^n \quad ; \quad \beta_k := \begin{cases} 0 & \text{for } k = 0 \\ \left((n-1)^{k-1} - \alpha_{k-1}\beta_{k-1}\right)^{1-n} & \text{else} \end{cases}$$

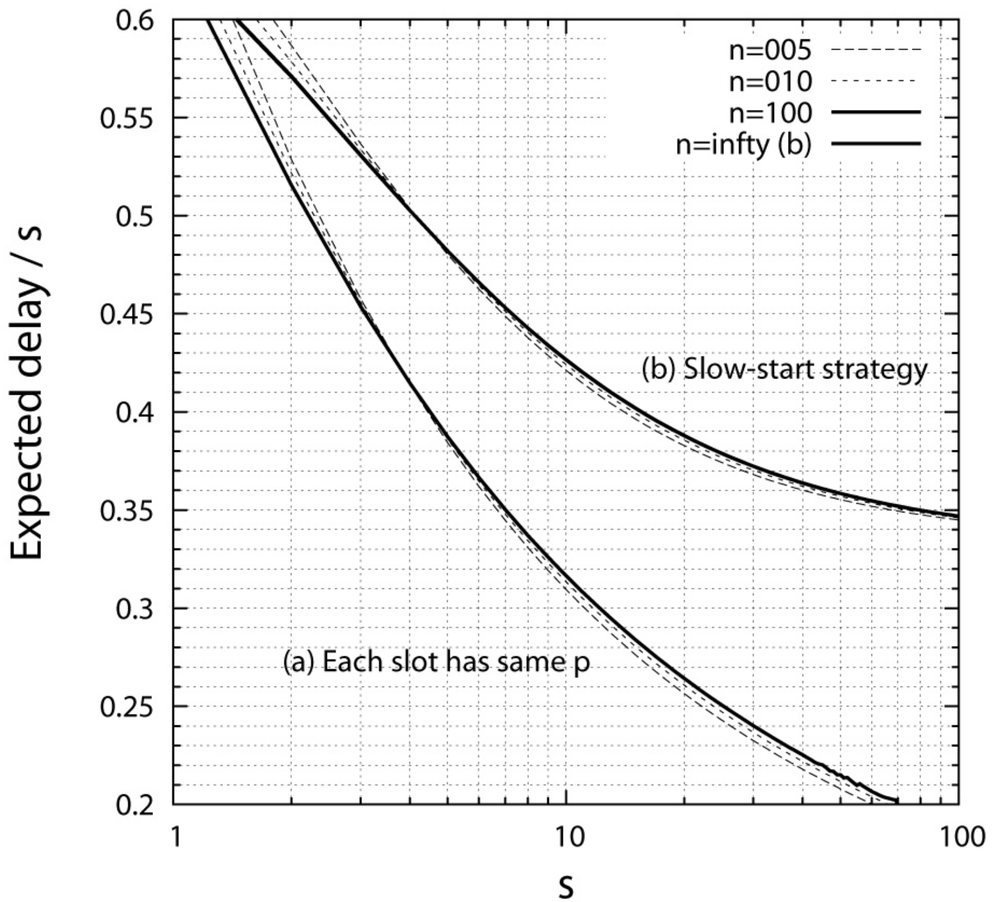
Important observation:

- A node is not forced to transmit within s slots ($\Sigma\, p_i \neq 1$)
- If we force each node to transmit within s slots, a worse probability is obtained.

Lakeside Labs

Delay of the first message

What is the expected delay of the first message? What is the maximum delay that can be guaranteed in 90% of all cases?



Lakeside Labs

Related work

- T. Watteyne, I. Augé-Blum, M. Dohler, D. Barthel: "Reducing collision probability in wireless sensor network backoff-based election mechanisms." In *Proc. IEEE GLOBECOM*, (Washington, DC), Nov. 2007.
- Y. Tay, K. Jamieson, H. Balakrishnan: "Collision-minimizing CSMA and its applications to wireless sensor networks." *IEEE J. Select. Areas Commun.*, Aug. 2004.
- J. A. Stine, G. de Veciana, K. H. Grace, R. D. Durst: "Orchestrating spatial reuse in wireless ad hoc networks using synchronous collision resolution." *J. Interconnection Networks*, Sept. 2002.

Conclusions and outlook

Analytical analysis of “non-colliding first messages” on a link with n nodes performing random access using ALOHA with s slots.

- The probability Φ that there occurs a non-colliding first message within the given slots is maximized by a **slow start strategy** of the nodes.
- We can calculate this optimal probability Φ and the **sending probabilities** p_i leading to this Φ, if we know n and s.
- The slow start strategy comes at the price of an **increased delay** of the first message; this delay is almost independent of n.

Outlook

- The number of nodes n is not always known. What is the sensitivity of Φ with respect to n? How can we estimate it?